

Total No. of Printed Pages: 2

**SUBJECT CODE NO: -BB-2421**  
**FACULTY OF SCIENCE AND TECHNOLOGY**  
**B.Sc (CBCGS)(Pattern 2022) S.Y SEM III**  
**Examination November / December 2024**  
**Mathematics-VI Laplace and Fourier Transforms**

[Time: 1:30 Hours]

[Max. Marks:40]

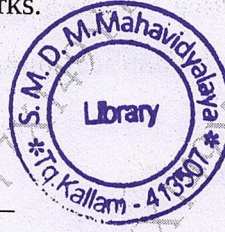
N. B

Please check whether you have got the right question paper.

- i. All questions are compulsory.
- ii. Figures to the right indicate full marks.

**Q.1 Choose the correct alternative.**

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(i) The value of  $\Gamma\left(\frac{1}{2}\right)$  is equal to \_\_\_\_\_

- |                           |                 |
|---------------------------|-----------------|
| a) $\frac{\pi}{2}$        | b) $\sqrt{\pi}$ |
| c) $\sqrt{\frac{\pi}{2}}$ | d) 0            |

(ii) If  $L\{f(t)\} = F(s)$ , then  $L\{e^{ut} f(t)\} =$  \_\_\_\_\_

- |              |              |
|--------------|--------------|
| a) $sF(s+a)$ | b) $sF(s-a)$ |
| c) $F(s+a)$  | d) $F(s-a)$  |

(iii)  $L^{-1}\left\{\frac{s}{s^2-a^2}\right\} =$  \_\_\_\_\_

- |            |            |
|------------|------------|
| a) cosh at | b) sinh at |
| c) sin at  | d) cos at  |

(iv) If  $F(s)$  is the complex Fourier transform of  $f(x)$ , then  $F\{f(ax)\} =$  \_\_\_\_\_

- |                                            |                                                |
|--------------------------------------------|------------------------------------------------|
| a) $\frac{1}{a} F\left(\frac{a}{s}\right)$ | b) $aF\left(\frac{s}{a}\right)$                |
| c) $\frac{1}{a} F\left(\frac{s}{a}\right)$ | d) $\frac{1}{a^2} F\left(\frac{s}{a^2}\right)$ |

(v) If  $F_s(s)$  and  $F_c(s)$  are Fourier sine and cosine transforms of  $f(x)$  respectively, then  $F_c\{f(x) \sin ax\} =$  \_\_\_\_\_

- |                                       |                                       |
|---------------------------------------|---------------------------------------|
| a) $\frac{1}{2}[F_c(s+a) + F_c(s-a)]$ | b) $\frac{1}{2}[F_s(s+a) + F_s(s-a)]$ |
| c) $\frac{1}{2}[F_c(s+a) - F_c(s-a)]$ | d) $\frac{1}{2}[F_s(s+a) - F_s(s-a)]$ |

**Q.2 (A) Attempt any one :**

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- a) If  $L\{f(t)\} = F(s)$ , then prove that  $L\left\{\frac{1}{t}f(t)\right\} = \int_s^\infty F(s)ds$ .
- b) If  $L\{f(t)\} = F(s)$ , then prove that
- $$L\{(f^{(n)}(t))\} = s^n F(s) - s^{n-1}f'(0) - s^{n-2}f''(0) - s^{n-3}f'''(0) - \dots - f^{(n-1)}(0) \text{ where } f^{(n)}(t) = \frac{d^n f(t)}{dt^n}.$$

**(B) Attempt any one :**

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- c) Find the Laplace transform of  $t^2 e^{-2t} \cos t$ .
- d) Find the inverse Laplace transform of  $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ .

**Q.3 (A) Attempt any one:**

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- a) If  $L^{-1}\{F(s)\} = f(t)$ , then prove that  $L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(t)dt$ .
- b) If  $F(s)$  is the complex Fourier transform of  $f(x)$ , then prove that
- $$F\{f(x) \cos ax\} = \frac{1}{2}[F(s+a) + F(s-a)].$$

**(B) Attempt any one :**

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- c) Using Laplace transform find the solution of the initial value problem  $y'' + y = \sin 3t$ , when  $y(0) = 0, y'(0) = 0$ .
- d) Find the Fourier sine and cosine transform of  $f(x) = e^{-ax}$ .

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