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SUBJECT CODE NO: - BB-2416

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc (CBCGS)(Pattern 2022) T.Y SEM V

Examination November / December 2024

Mathematics-XV Real Analysis-I

[Time: 1:30 Hours]

[Max. Marks: 40]

Please check whether you have got the right question paper.

N. B

1) All questions are compulsory.

2) Figures to the right indicate full marks.

Q.1 Attempt Any Five

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- (a) Let $f: R \rightarrow R$, defined by $f(x) = e^x$ then show that f is not onto function.
- (b) Consider the function defined by $f(x) = \sin x$, $(-\infty < x < \infty)$, then find $f^{-1}(1)$.
- (c) Suppose $\{s_n\}_{n=1}^{\infty}$ diverges to infinity then prove that $\{(-1)^n s_n\}_{n=1}^{\infty}$ oscillates.
- (d) Show that the sequence $\{n\}_{n=1}^{\infty}$ diverges to infinity.
- (e) If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B then prove that $\sum_{n=1}^{\infty} (a_n - b_n)$ converges to $A - B$.
- (f) Every conditionality convergence series is not convergent. Explain with reason.
- (g) If $u_1, u_2, \dots, u_n$ are functions of $x_1, x_2, \dots, x_n$ and $x_1, x_2, \dots, x_n$ are functions of $y_1, y_2, \dots, y_n$ then $\frac{\partial u_1}{\partial y_1} = \dots$ and $\frac{\partial(u_1, u_2, \dots, u_n)}{\partial(x_1, x_2, \dots, x_n)} = \dots$



Q.2 (A) Attempt Any One

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- (a) Let $f: A \rightarrow B$, $g: B \rightarrow C$, and $h: C \rightarrow D$ be functions. Prove that the following statement holds:

$$ho(gof) = (hog)of$$

- (b) If $\sum_{n=1}^{\infty} a_n$ is a series of non-negative numbers with $s_n = a_1 + \dots + a_n$ ($n \in I$), then prove that
- $\sum_{n=1}^{\infty} a_n$ converges if the sequence $\{s_n\}_{n=1}^{\infty}$ is bounded.
 - $\sum_{n=1}^{\infty} a_n$ diverges if $\{s_n\}_{n=1}^{\infty}$ is not bounded.

(B) Attempt Any One.

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- (a) Show that the set of all integers is countable.
- (b) Prove that the series $(a_1 - a_2) + (a_2 - a_3) + (a_3 - a_4) + \dots$ converges if and only if the sequence $\{a_n\}_{n=1}^{\infty}$ converges.

