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SUBJECT CODE NO: - BB-2417
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc (CBCGS)(Pattern 2022) T.Y SEM V
Examination November / December 2024
Mathematics-XVI Abstract Algebra

[Time: 1:30 Hours]

[Max. Marks: 40]

Please check whether you have got the right question paper

Q.1 Attempt any five

10

- Show that inverse of an element of group G is unique.
- Find all the right cosets of subgroup $H = \{e, \Psi, \Psi^2\}$ in the group S_3 .
- Given a, b in the group G then prove that the equation $a \cdot x = b$ have solution for x in G
- Let G be the group of integers under addition and let $\bar{G} = G$. Also $\phi: G \rightarrow \bar{G}$ is defined by $\phi(x) = 2x$ for all $x \in G$ then find kernel of ϕ .
- If ϕ is a homomorphism of ring R into ring R^1 then prove that $\phi(0) = 0$.
- If R is a commutative ring and $a \in R$ then show that $aR = \{ar/r \in R\}$ is a left ideal of R .
- Define i) Field ii) Integral domain

Q.2 (A) Attempt any one:

8

- Prove that a homomorphism ϕ of G into $\bar{G}$ with Kernel K_ϕ is an isomorphism of G into $\bar{G}$ if and only if $K_\phi = (e)$.
- Prove that a Subgroup N of a group G is a normal subgroup if and only if Product of two right cosets of N in G is again a right coset of N in G .

(B) Attempt any one:

7

- If G is a group and $a \in G$ then show that $N(a) = \{x \in G \mid xa = ax\}$ is a Subgroup of G .
- Show that the set of all non zero rational numbers forms a group under usual multiplication.

Q.3 (A) Attempt any one:

8

- (a) Prove that every finite integral domain is a field.
- (b) If two polynomials $f(x)$ and $g(x) \neq 0$ in $F[x]$ then prove that there exist polynomials $t(x)$ and $r(x)$ such that $f(x) = t(x)g(x) + r(x)$, $\deg(r(x)) < \deg(g(x))$.

(B) Attempt any one:

7

- (a) Prove that intersection of two ideals of R is also an ideal of R .
- (b) Prove that $f(x) = x^2 + 1$ is irreducible over the field of integers modulo 7.
