

SUBJECT CODE NO: B-2118
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. Pattern - 2013 (T.Y SEM V)
Examination November / December 2025
Mathematics MAT - 501 Real Analysis - I

[Time: 1:30 Hours]

[Max. Marks: 50]

Please check whether you have got the right question paper.

N. B

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1

a) Attempt any one of the following:

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- i) If $u_1, u_2, \dots, u_n$ are implicit function of $x_1, x_2, \dots, x_n$, then with usual notations prove that

$$\frac{\partial(F_1, F_2, \dots, F_n)}{\partial(u_1, u_2, \dots, u_n)} \cdot \frac{\partial(u_1, u_2, \dots, u_n)}{\partial(x_1, x_2, \dots, x_n)} = (-1)^2 \frac{\partial(F_1, F_2, \dots, F_n)}{\partial(x_1, x_2, \dots, x_n)}$$

- ii) If $f: A \rightarrow B$ and if $X \subseteq B, Y \subseteq B$, then prove that
 $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$.

b) Attempt any one of the following:

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- i) If the sequence $\{s_n\}_{n=1}^{\infty}$ is divergent sequence of real numbers and $c \in \mathbb{R}, c \neq 0$, then prove that $\{cs_n\}_{n=1}^{\infty}$ diverges.

- ii) If the roots of the equation $\lambda^3 - (\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ are u, v, w , then prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -2 \frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$$

Q2

a) Attempt any one of the following:

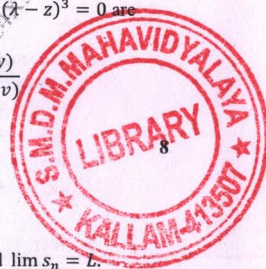
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- i) If $\{s_n\}_{n=1}^{\infty}$ is a sequence of the real numbers, and if

$$\lim_{n \rightarrow \infty} \inf s_n = \lim_{n \rightarrow \infty} \sup s_n = L,$$

Where $L \in \mathbb{R}$, then prove that $\{s_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n \rightarrow \infty} s_n = L$.

- ii) If the series $\sum_{n=1}^{\infty} a_n$ is convergent series, then prove that $\lim_{n \rightarrow \infty} a_n = 0$.



b) Attempt any one of the following:

7

- Prove that if $\{s_n\}_{n=1}^{\infty}$ converges to 0, then $\{s_n\}_{n=1}^{\infty}$ converges to 0.
- Prove that if $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} (a_n + b_n)$ diverges.

Q3

a) Attempt any one of the following:

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- Prove that the set $\mathbb{R}$ of all real numbers is uncountable.
- If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.

b) Attempt any one of the following:

5

- For what values of p the series $\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots$ converges?
- Show that the sequence $\left\{ \log \frac{1}{n} \right\}_{n=1}^{\infty}$ diverges to minus infinity.

Q4 Choose the correct alternative and rewrite the sentence.

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a) If f and g are two real-valued functions, then $\min(f, g) =$

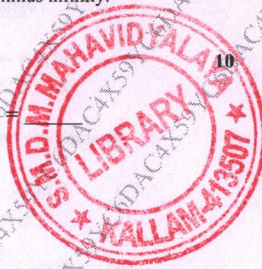
- $\frac{|f-g|+f+g}{2}$
- $\frac{-|f-g|+f+g}{2}$
- $\frac{|f-g|-f-g}{2}$
- $\frac{-|f-g|-f-g}{2}$

b) A sequence $\{s_n\}_{n=1}^{\infty}$ of real numbers is a function from the set of positive integers into the _____

- Set of integers
- Set of real numbers
- Set of rational numbers
- Set of irrational numbers

c) If $\{t_n\}_{n=1}^{\infty}$ is a sequence of real numbers, and if $\lim_{n \rightarrow \infty} t_n = M$, then $\lim_{n \rightarrow \infty} \frac{1}{t_n} =$

- M
- $|M|$
- $\frac{1}{M}$
- $\frac{1}{|M|}$



d) The series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$ is _____

- i. Both convergent and absolutely convergent
- ii. Convergent but not absolutely convergent
- iii. Not convergent and not absolutely convergent
- iv. Divergent but absolutely convergent

e) If $u = r \cos \theta$, $v = r \sin \theta$, then $\frac{\partial(u,v)}{\partial(r,\theta)} =$ _____

- i. $\cos \theta$
- ii. $\sin \theta$
- iii. r
- iv. r

