

SUBJECT CODE NO: BB-2332
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. (CBCGS)(Pattern 2022) T.Y SEM V
Examination November / December 2025
Mathematics-XV Real Analysis-I

[Time: 1.30 Hours]

[Max. Marks:40]

Please check whether you have got the right question paper.

N. B

1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q1 Attempt Any Five**10**

a) Show that composition of two one-to-one mappings is one-to-one.

b) Let

$$f(x) = \log x, 0 < x < \infty$$

Then what is the range of f ?c) Find $N \in \mathbb{I}$ so that $\frac{1}{\sqrt{n+1}} < 0.003$ when $n \geq N$.

d) Explain with reason that every bounded sequence is not convergent.

e) If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B then prove that

$$\sum_{n=1}^{\infty} (a_n + b_n) \text{ converges to } A + B.$$

f) Define (i) Conditional Convergence. (ii) Absolute convergence

g) If $u_1 = f_1(x_1), u_2 = f_2(x_1, x_2), \dots, u_n = f_n(x_1, x_2, \dots, x_n)$, then find $\frac{\partial(u_1, u_2, \dots, u_n)}{\partial(x_1, x_2, \dots, x_n)}$ **Q2****A. Attempt Any One****08**

a) Define

- i. Bounded above set.
- ii. Bounded below set.
- iii. Least upper bound.
- iv. Greatest lower bound.

b) If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then prove that $\sum_{n=1}^{\infty} a_n$ converges.**B. Attempt Any One****07**c) Consider the sine function defined by $f(x) = \sin x$ ($-\infty < x < \infty$).

- i. What is the image of $\frac{\pi}{2}$ under f ?
- ii. Find $f^{-1}(1)$.
- iii. Find $f([0, \frac{\pi}{6}])$, $f([\frac{\pi}{6}, \frac{\pi}{2}])$ and $f([0, \frac{\pi}{2}])$.
- iv. Let $A = [0, \frac{\pi}{6}]$ and $B = [\frac{5\pi}{6}, \pi]$. Does $f(A \cap B) = f(A) \cap f(B)$.

d) Show that $\sum_{k=1}^{\infty} a_k$ converges if and only if given $\epsilon > 0$, there exists $N \in \mathbb{I}$ such that $|\sum_{k=m+1}^n a_k| < \epsilon$ ($n, m \geq N$)

Q3

A. Attempt Any One

08

- a) Show that every Cauchy sequence of real numbers is a convergent sequence.
 b) For any $a, b \in \mathbb{R}$, show that $||a| - |b|| \leq |a - b|$.

Also prove that $\{|s_n|\}_{n=1}^{\infty}$ converges to $|L|$ if $\{s_n\}_{n=1}^{\infty}$ converges to L .

B. Attempt Any One

07

- c) For $n \in \mathbb{I}$, let

$$t_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!},$$

then prove that $\{t_n\}_{n=1}^{\infty}$ is convergent.

- d) If $u = x^2 + y^2 + z^2$, $v = x + y + z$, $w = xy + yz + zx$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

