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SUBJECT CODE NO: -BB-2333
FACULTY OF SCIENCE AND TECHNOLOGY
B. Sc. (CBCGS) (Pattern 2022) T.Y SEM V
Examination November / December 2025
Mathematics-XVI Abstract Algebra

[Time: 1:30 Hours]**[Max. Marks: 40]**

Please check whether you have got the right question paper.

N. B

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1 Attempt any five**10**

1. If G is a group then prove that for every $a \in G$, $(a^{-1})^{-1} = a$
2. Find all the right cosets of $H = \{e, \phi, \psi\}$ in the group S_3 .
3. Given a, b in the group G , then prove that the equation $y \cdot a = b$ have solution for Y in G .
4. Let G is a group of real numbers under addition and $\bar{G} = \bar{G}$. Also $\phi: G \rightarrow \bar{G}$ is defined by

$$\phi(x) = 13x \text{ for all } x \in G$$

Show that ϕ is a homomorphism.

5. If ϕ is a homomorphism of ring R into ring R' then prove that $\phi(-a) = -\phi(a)$
 For every $a \in R$
 If R is a ring and $a \in R$ then show that $Ra = \{xa | x \in R\}$ is a left ideal of R .

6. Define

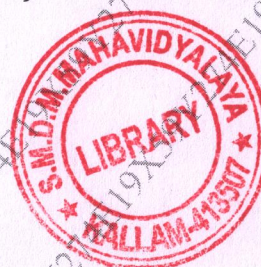
- i. Characteristics zero of an integral domain
- ii. Finite characteristic of an integral domain.

Q2 A] Attempt any one:**08**

- a) State and Prove Lagrange's theorem.
- b) If G is a group and $Z = \{X \in G | ZX = XZ, \text{ for all } X \in G\}$ then prove that Z is a subgroup of G .

B] Attempt any one**07**

- a) If the mapping ϕ is a homomorphism from the group G into the group $\bar{G}$ then prove that
 - i. $\phi(e) = \bar{e}$
 - ii. $\phi(a^{-1}) = [\phi(a)]^{-1}$, for $a \in G$.
- b) If the index of a subgroup H in G is 2 then prove that H is a normal subgroup of G .



Q3 A] Attempt any one:

08

- a) Prove that every field is an integral domain.
- b) If F is a field, then show that its only ideals are $\{0\}$ and F itself.

B] attempt any one:

07

- a) If $f(x), g(x)$ are two non-zero polynomials of the ring of polynomials $F[x]$ then prove that. $\deg(f(x) \cdot g(x)) = \deg(f(x)) + \deg(g(x))$
- b) Show that the set of all even integers is a ring with respect to usual addition and usual multiplication.

