

Total No. of Printed Pages:2

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FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. Pattern - 2013 (T.Y SEM VI)
Examination November / December 2025
Mathematics MAT-601 Real Analysis-II

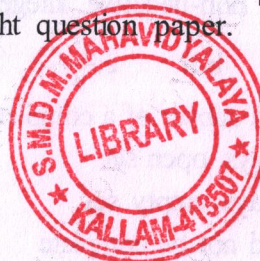
[Time:1:30 Hours]

[Max. Marks:50]

N. B

Please check whether you have got the right question paper.

- 1) All questions are compulsory.
- 2) Figure to right indicate full marks.

**Q1 a) Attempt any one:****08**

1. Define open set in a metric space. Prove that in any metric space (M, ζ) , both M and $\emptyset$ are open sets.
2. If $\mathcal{F}$ is any family of closed subsets of a metric space M , then prove that $\bigcap F$ is also closed, $F \in \mathcal{F}$

b) Attempt any one:**07**

3. If $n \in \mathbb{I}$ fixed. If $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ are two ordered n -tuples of real numbers, define $\zeta: \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ by the rule:

$$\zeta(x, y) = \left[\sum_{k=1}^n (x_k - y_k)^2 \right]^{1/2}$$

4. If A and B are closed subsets of $\mathbb{R}$ then prove that $A \times B$ is closed subset of $\mathbb{R}^2$.

Q2 a) Attempt any one**08**

- 1) If $f(x)$ is Riemann-integrable in every interval and is periodic with period 2π , then prove that

$$\int_{-\pi}^{\pi} f(x) dx = \int_{-\pi}^{\pi} f(a+x) dx$$

where a is any number.

- 2) If $f \in R[a, b]$ and $a < c < b$, then prove that $f \in R[a, c]$, $f \in R[c, b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$.

b) Attempt any one:**07**

- 3) Prove that every finite subset of any Metric space is compact.
- 4) Obtain the Fourier series expansion of $f(x) = x \sin x$, in $[-\pi, \pi]$.

Q3 a) Attempt any one:**05**

- 1) Let A be a subset of the metric space (M, ζ) . If (A, ζ) is compact in A , then prove that A is closed subset of (M, ζ) .
- 2) If each of subset $E_1, E_2, \dots$ of $\mathbb{R}^1$ is of measure zero, then prove that $\bigcup_{n=1}^{\infty} E_n$ is also of measure zero

b) Attempt any one:**05**

- 3) Let $f(x) = x$, $(0 \leq x \leq 1)$ and σ be the subdivision $\{0, 1/3, 2/3, 1\}$ of $[0, 1]$. Compute $L[f; \sigma]$.

4) If $T: X \rightarrow X$ is defined as $Tx = x^2$; ($0 \leq x \leq 1/3$), prove that T is contraction on $[0, 1/3]$.

Q4 Choose the correct alternative and rewrite the sentence.

10

1. For all non-negative integer n , $\int_{-\pi}^{\pi} w s^2 n x dx = \dots$

- a. $\frac{\pi}{2}$ b. π c. $\frac{\pi^2}{4}$ d. π^2

2. Closed subset of a complete metric space is ---

- a. always open set
b. always empty set
c. need not be complete
d. complete

3. If M is closed interval $[0, 1]$ with absolute value metric, then $B\left[\frac{1}{4}, \frac{1}{2}\right]$ is ---

- a. $\left[0, \frac{3}{4}\right)$ b. $\left[-\frac{1}{2}, \frac{1}{2}\right)$ c. $\left[-\frac{1}{4}, \frac{1}{4}\right)$ d. $\left[-\frac{3}{2}, \frac{3}{4}\right)$

4. The set of cluster points of $(0, 1)$ in $\mathbb{R}^d$ is ---

- a. $(0, 1)$ b. $[0, 1]$ c. $\mathbb{R}$ d. empty set

5. Every singleton set in a discrete metric space $\mathbb{R}^d$ is ---

- a. closed
b. open
c. open and closed
d. None of these.

