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**SUBJECT CODE NO: - B-2185**  
**FACULTY OF SCIENCE AND TECHNOLOGY**  
**B.Sc. Pattern - 2013 (T.Y SEM VI)**  
**Examination November / December 2025**  
**Mathematics MAT - 602 Abstract Algebra - II**

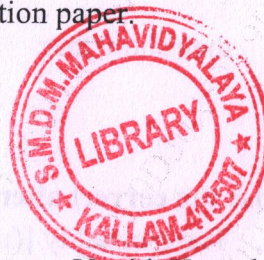
[Time: 1:30 Hours]

[Max. Marks:50]

Please check whether you have got the right question paper.

N. B

- 1) All questions are compulsory.
- 2) Draw diagram whenever necessary.
- 3) Use Only blue or black pen for writing.

**Q1 A) Attempt any one:**

08

1. If  $T$  is homomorphism of a vector space  $U$  onto a vector space  $V$  with Kernel  $W$ , then prove that  $V$  is isomorphic to  $U/W$ .
2. If  $v_1, v_2, \dots, v_n$  are in vector space  $V$ , then prove that either they are linearly independent or some  $V_k$  is a linear combination of the preceding ones  $v_1, v_2, \dots, v_{k-1}$ .

**B) Attempt any one:**

07

1. Prove that the sum of two subspaces of a vector space is a subspace.
2. If  $F$  is a field of real numbers, then prove that the vectors  $(1, 2, 0, 0), (0, 1, -1, 0)$  and  $(0, 0, 0, 3)$  in  $F^{(4)}$  are linearly independent over  $F$ .

**Q2 A) Attempt any one:**

08

1. If  $V$  is finite-dimensional vector space and  $W$  is subspace of  $V$ , then prove that  $A(A(W)) = W$ .
2. If  $W$  is subspace of a vector space  $V$  then prove that  $W^\perp$  is subspace of  $V$ .

**B) Attempt any one:**

07

1. If  $W_1$  and  $W_2$  are the subspaces of a finite-dimensional vector space  $V$ , then show that:

$$A(W_1 \cap W_2) = A(W_1) + A(W_2).$$

2. Prove that the intersection of two submodules of a module  $M$  is a submodule of  $M$ .

**Q3 A) Attempt any one:**

05

1. If  $a, b, c$  are real numbers such that  $a > 0$  and  $a\lambda^2 + 2b\lambda + c \geq 0$  for all real number  $\lambda$ , then prove that  $b^2 \leq ac$ .
2. If  $V$  is a finite-dimensional inner product space and  $W$  is a subspace of  $V$ , then prove that  $(W^\perp)^\perp = W$ .

**B) Attempt any one:**

1. If  $T$  is a homomorphism of a module  $M$  into a module  $N$ , then prove that  $I(T) = \{xT \mid x \in M\}$  is a submodule of  $N$ .
2. If  $V$  the set of all continuous complex valued function on  $[0,1]$  with inner product defined by

$$(f(t), g(t)) = \int_0^1 f(t) \overline{g(t)} dt,$$

then show that:

$$|(f(t), g(t))| \leq \int_0^1 |f(t)|^2 dt \int_0^1 |g(t)|^2 dt.$$

**Q4 Choose the correct alternative and rewrite the sentence.**

1. The inequality  $|(u, v)| \leq \|u\| \|v\|$  is known as \_\_\_\_\_.  
 a) Bessel's inequality                      b) Schwarz inequality  
 c) Triangle inequality                      d) None of these
2. If  $U$  and  $V$  are vector space over the Field  $F$ , if  $T: U \rightarrow V$  is homomorphism, then the Kernel of  $T$  is given by  $\text{Ker}(T) =$  \_\_\_\_\_.  
 a)  $\{u \in V / T(u) = 0\}$                       b)  $\{u \in V / T(u) = v\}$   
 c)  $\{u \in U / T(u) = u\}$                       d)  $\{u \in U / T(u) = 0\}$
3. If  $T$  is a homomorphism of module  $M$  into  $N$ , then  $T$  is isomorphism if \_\_\_\_\_.  
 a)  $K(T) = (0)$                       b)  $K(T) = T$   
 c)  $K(T) = M$                       d)  $K(T) = N$
4. Let  $W$  be a subspace of vector space  $V$  then  $W \cap W^\perp =$  \_\_\_\_\_.  
 a)  $(0)$                       b)  $(1)$                       c)  $\phi$                       d) None of these
5. If  $T$  is a homomorphism of  $U$  onto  $V$  with Kernel  $W$ , then  $V$  is isomorphic to \_\_\_\_\_.  
 a)  $\frac{V}{W}$                       b)  $\frac{U}{W}$                       c)  $\frac{W}{U}$                       d)  $\frac{V}{U}$