Total No. of Printed Pages: 3

SUBJECT CODE NO:- B-2039
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. F.Y. (Sem-I)
Examination November/December- 2022
Mathematics MAT – 101
Differential Calculus

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B

1) Attempt all question
2) Figure to the right indicate full marks.

Q. 1 (A) Attempt any one: 08

(a) If U and V be two functions of x possessing derivatives of the nth order then prove that, $(UV)_n = U_n + nC_1\, U_{n-1}\, V_1 + nC_2\, U_{n-2}\, V_2$ +________ +_________+ $nC_r\, U_{n-r}\, V_r$ +__________+ ${}^nC_n\, UV_n$

(b) Show that, if fis finitely derivable at c, then f is also continuous at c.

(B) Attempt any one: 07

(c) If $f(x)=x^2 \sin(1/x)$ when x≠o and f(o), show that f is derivable for every value of x but the derivative is not continuous for x=o

(d) If y= a cos (log x)+b sin (log x). show that,

$$x^2 \frac{d^2y}{dx^2} + x\,\frac{dy}{dx} + y = 0$$

Q. 2 (A) Attempt any one: 08

(a) If a function f is,

i. Continuous in closed interval [a,b]
ii. Derivable in the open interval (a,b)
iii. f(a)=f(b), Then, Prove that, three exists at least one value c∈ (a, b) such that, $f^1(c)=0$

(b) If $z=f(x_1y)$ is homogeneous function of x,y of degree n, then prove that,

$$x^2\,\frac{\partial^2 z}{\partial x^2} + 2xy\,\frac{\partial^2 z}{\partial x\,\partial y} + y^2\,\frac{\partial^2 z}{\partial y^2} = n(n-1)z$$

(B) Attempt any one:

(c) Discuss applicability of Rolle's theorem to the function f(x) = $|x|\ in\ [-1,1]$ 07

(d) If z=(x+y) ∅ ($\frac{y}{x}$), where ∅ is any x $\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$

Q. 3 (A) Attempt any one: 05

(a) Prove that, the gradient of scalar point function is a vector point function

(b) Prove that, grad f(r) x $\underset{r}{\rightarrow}$= 0 where, $r = \sqrt{x^2 + y^2 + z^2}$ and $\underset{r}{\rightarrow} = x\bar{\imath} + y\bar{\imath} + z\bar{k}$

(B) Attempt any one: 05

(c) Show that,

Grad $(\underset{f}{\rightarrow}.\underset{g}{\rightarrow}) = \underset{f}{\rightarrow}\times\ curl\ \underset{g}{\rightarrow} + \underset{g}{\rightarrow}\times\ curl\ \underset{f}{\rightarrow} + \left(\underset{f}{\rightarrow}\nabla\right)\underset{g}{\rightarrow} + (\underset{g}{\rightarrow}.\nabla)\underset{f}{\rightarrow}$

(d) Show that ∀ $x \in R$

Sin x=$x - \frac{x^3}{3!} + \frac{x^5}{5!} +$ ________

Q. 4 Choose the correct alternative. 10

i. $\lim_{x\to o} \sin(\frac{1}{x})$ ________

a) Exists

b) Is equal to zero

c) Is equal to ∞

d) Does not exists

ii. If $x^p\ y^q = (x+y)^{p+q}$ Then, $\frac{dy}{dx}$ is equal to________

a) $\frac{y}{x}$

b) $\frac{py}{qx}$

c) $\frac{x}{y}$

d) $\frac{qy}{px}$

iii. If x=t-sin t, y=1-cost, Then $\frac{d^2y}{dx^2}$ at $(\pi, 2)$ will be_______

a) 0

b) 1

c) π

d) ∞

iv. If f is continuous in [a,b] and differentiable in (a,b) then three exists at least one point C in (a,b) such that $f^1(c)$ is equal to_________

a) $\frac{f(b)+f(a)}{b+a}$

b) $\frac{f(b)-f(a)}{b+a}$

c) $\frac{f(b)-f(a)}{b-a}$

d) $\frac{f(b)+f(a)}{b-a}$

v. curl $\underset{r}{\rightarrow}$=______

a) 1

b) 2

c) 3

d) 0