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SUBJECT CODE NO: - CB-2340
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y. (Sem-I)
Examination December/January-2022-23
Mathematics -II MAT - 021 Deferential Calculus

[Time: 1.30 Hours]

[Max. Marks:40]

Please check whether you have got the right question paper.

N. B

- 1) Attempt all questions
- 2) Figures to the right indicate full marks.

Q1

A) Attempt any one

05

- a) Prove that the union of an arbitrary family of open sets is open.
- b) Prove that every convergent sequence is bounded.

B) Attempt any one:

05

- a) Show that $\lim_{n \rightarrow \infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$
- b) Examine the continuity of the function

$$f(x) = \begin{cases} \frac{\sin(x-c)}{(x-c)}, & \text{if } x \neq c \\ 0, & \text{if } x = c \end{cases}$$

at $x = c$

Q2

A) Attempt any one

05

- a) If $y = \sin(ax + b)$, then prove that $\frac{d^n y}{dx^n} = a^n \sin\left(ax + b + n\frac{\pi}{2}\right)$
- b) If a function f defined on $[a, b]$ is continuous on $[a, b]$ and derivable on $] a, b [$, then prove that there exist at least one real number C between a and b such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

B) Attempt any one

05

- c) If $y = \log(\sin x)$, then prove that $y_3 = \frac{2 \cos x}{\sin^3 x}$
- d) Discuss the derivability of the function $f(x) = |x| + |x - 1|$ at $x=0$

Q3

A) attempt any one

05

- a) If ϕ and ψ are two scalar point functions then prove that $\text{grad}(\phi + \psi) = \text{grad} \phi + \text{grad} \psi$
- b) If ϕ is a scalar point function and \vec{u} is a vector point function then prove that $\text{div}(\phi \vec{u}) = (\text{grad} \phi) \cdot \vec{u} + \phi(\text{div} \vec{u})$

B) Attempt any one

c) If $\phi(x, y, z) = 3x^2y - y^3z^2$ find grad ϕ at the point $(1, -2, -1)$

05

d) Find $\frac{d^n y}{dx^n}$, if $y = x^3 \cos x$

Q4 Choose the correct alternative

10

1) If $S_n = (-1)^n, n \in N$, then upper limit of $S_n =$ -----

- a) -1 b) 1 c) 0 d) ∞

2) Limit point of the set $\left\{\frac{1}{n}, n \in N\right\}$ is -----

- a) 0 b) -1 c) 1 d) $\frac{1}{2}$

3) The vector differential operator del (∇) = -----

- a) $\vec{i} dx + \vec{j} dy + \vec{k} dz$ b) $\partial x + \partial y + \partial z$
 c) $\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}$ d) $\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$

4) If $y = \frac{\log x}{x}$, then $\frac{d^2 y}{dx^2} =$ -----

- a) $\frac{2 \log x}{x}$ b) $\frac{\log x}{2x}$ c) $2 \log x + 3$ d) $\frac{2 \log x - 3}{x^3}$

5) $\lim_{x \rightarrow 0^-} \frac{|x|}{x} =$ -----

- a) -1 b) 0 c) 1 d) does not exist