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**SUBJECT CODE NO:- B-2050**  
**FACULTY OF SCIENCE & TECHNOLOGY**  
**B.Sc. S.Y. (Sem-III)**  
**Examination November/December- 2022**  
**Mathematics MAT – 301**  
**Number Theory**

[Time: 1:30 Hours]

[Max. Marks: 50]

Please check whether you have got the right question paper.

- N.B
- i) All questions are compulsory.
  - ii) Figures to the right indicate full marks.
- Q.1
- a) Attempt any one of the following: 08
    - i. If  $k > 0$ , then prove that  $\gcd(ka, kb) = k \gcd(a, b)$ .
    - ii. For integers  $a, b, c$ , prove the following
      - $\alpha$ ) if  $a|b$  and  $b|c$  then  $a|c$ ,
      - $\beta$ ) if  $a|b$  and  $a|c$  then  $a|(bx + cy)$  for arbitrary integers  $x$  and  $y$
  - b) Attempt any one of the following: 07
    - i. If  $a$  is odd integer, then prove that  $32|(a^2 + 3)(a^2 + 7)$ .
    - ii. Find all solutions in the integers of the Diophantine equation  $24x + 138y = 18$ .
- Q.2
- a) Attempt any one of the following: 08
    - i. State and prove Chinese remainder theorem.
    - ii. If  $p$  is prime number, then prove that  $(p-1)! \equiv -1 \pmod{p}$ .
  - b) Attempt any one of the following: 07
    - i. Solve the linear congruence  $25x \equiv 15 \pmod{29}$ .
    - ii. If  $\gcd(a, 133) = \gcd(b, 133) = 1$ , then show that  $133 | a^{18} - b^{18}$ .

Q.3 a) Attempt any one of the following:

05

- i. If  $p$  is a prime number and  $p|ab$ , then prove that  $p|a$  or  $p|b$ .
- ii. If  $F$  is multiplicative function and is defined by

$$F(n) = \sum_{d|n} f(d),$$

then prove that  $f$  is multiplicative function.

b) Attempt any one of the following:

05

- i. Calculate  $\phi(360)$ .
- ii. Find the values of  $\tau(180)$  and  $\sigma(180)$ .

Q.4 Choose the correct alternative and rewrite the sentence:

10

1)  $\gcd(-12,30) = \text{-----}$

- a) 6      b) 4      c) 3      d) 1

2) The number of solutions of linear congruence  $6x \equiv 15 \pmod{21}$  is ..

- a) 6      b) 3      c) 1      d) 15

3) The value of  $\mu(10)$  is -----

- a) -1      b) 0      c) 5      d) 1

4) If  $\gcd(a, b) = d$ , then  $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = \text{-----}$

- a) 1      b)  $d$       c)  $\frac{1}{d}$       d)  $ab$

5) If  $a|bc$  with  $\gcd(a, b) = 1$  then -----

- a)  $b|a$       b)  $a|c$       c)  $c|a$       d)  $a = b$