Total No. of Printed Pages: 2

SUBJECT CODE NO:- B-2051
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. S.Y. (Sem-III)
Examination November/December- 2022
Mathematics MAT - 302
Integral Transforms

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B All questions are compulsory ,between internal choice in available
Figures to the right indicate full marks

Q.1 (a) Attempt any one of the following: 08

i. If L^{-1}\{f(s)\} = F(t), then prove that $(L^{-1})f^{n}$(s)\} = $(-1)^{n}t^{n}F(t)$

ii. Derive the relation between Fourier transform and Laplace transform.

(b) Attempt any one of the following: 07

i. Using Laplace transform, find the solution of the differential equation $(D^2 + D)y = t^2 + 2t$, where y(0)=4 $y'(0) = -2$

iii. Find the value of $L^{-1}\left\{\frac{1}{s(s+1)^3}\right\}$

Q.2 a) Attempt any one of the following 08

i. If L\{F(t)\}=f(s),then prove that $\lim_{s\to\infty} F(t) =.\lim_{t\to 0} sf(s)$

ii. If $\tilde{f}\,(s)$and $\tilde{g}$ (s) are Fourier transforms of f(x)and g(x) respectively, then prove that

$F\{a\,f(x) + bg(x)\}$ = a $\tilde{f}\,(s) + b\,\tilde{g}$ (s)

Where a and b are constants

b) Attempt any one of the following 07

i. Prove that $L^{-1}\left\{\tan^{-1}\frac{2}{s^2}\right\} = \frac{2}{t}\ sintsinh\,t.$

ii. Using Laplace transform, prove that $\int_0^{\infty} te^{-3t}sintdt = \frac{3}{50}$

Q.3 (a) Attempt any one of the 05

i. If L\{F(t)\} = f(s), then prove that L\{e^{at}F(t)\} = f(s + a).

ii. If f(s) is the Fourier transform F(x), then prove that the Fourier transform of F'(x) is equal to is f(s).

(b) Attempt any one of the following: 05

i. Evaluate the integral

$$\int_0^{\infty} e^{ax} x^{m-1} sinbxdx$$

ii. Evaluate L{sin at - at cos at}.

Q.4 Choose the correct alternative and rewrite the sentence: 10

(a) If $\int_0^{\infty} e^{-} dx = \frac{\sqrt{\pi}}{2}, then \int_{-\infty}^{\infty} e^{-x^2 dx} =$______

i. $\frac{\sqrt{\pi}}{2}$

ii. $\sqrt{\frac{\pi}{2}}$

iii. $\sqrt{\pi}$

iv. 0

(b) $L\{2t^3 - 6t + 8\} =$______

i. $\frac{12}{s^3} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

ii. $\frac{6}{s^4} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

iii. $\frac{12}{s^4} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

iv. $\frac{12}{s^4} - \frac{6}{s^2} + \frac{8}{s}, s > 0$

(c) $L^{-1}\left\{\frac{1}{s-a}\right\} =$____$, s > a$

i. ae^t

ii. a^{at}

iii. a^{-at}

iv. ae^{-t}

(d) The sine transform of f(x)=$\frac{1}{x}$ is____________

i. $\sqrt{\pi}$

ii. π

iii. 2π

iv. $\frac{\pi}{2}$

(e) $L\{\sinh at\} =$______

i. $\frac{a}{s^2-a^2}$

ii. $\frac{s}{s^2-a^2}$

iii. $\frac{a}{s^2+a^2}$

iv. $\frac{s}{s^2+a^2}$