Total No. of Printed Pages: 3

SUBJECT CODE NO:- B-2046
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T. Y. (Sem-V)
Examination November/December- 2022
Mathematics MAT - 501
Real Analysis – I

[Time: 1:30 Hours] **[Max. Marks:50]**

"Please check whether you have got the right question paper."

N.B.
1) All questions are compulsory.
2) Figures to the right indicate Full marks.

Q.1 A] Attempt any one:

a) Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent. 08

b) Define Cauchy sequence. 08
If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges then prove that $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

B] Attempt any one:

c) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers diverging to infinity, then prove that $\lim_{n\to\infty} \operatorname{Sup} S_n = \infty = \lim_{n\to\infty} \inf S_n$. 07

d) For $n \in I$, let $S_n = \frac{1\cdot 3\cdot 5\cdot\ldots(2n-1)}{2\cdot 4\cdot 6\cdot\ldots 2n}$ 07
Prove that $\{S_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} S_n \le \frac{1}{2}$.

Q.2 A] Attempt any one:

a) Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero real numbers and let $a = \lim_{n\to\infty} \inf\left|\frac{a_{n+1}}{a_n}\right|$, 08
$A = \lim_{n\to\infty} \operatorname{Sup}\left|\frac{a_{n+1}}{a_n}\right|$ Then prove that $\sum_{n=1}^{\infty}|a_n| < \infty$ if $A < 1$.

b) If $\sum_{n=1}^{\infty} a_n$ is a divergent series of positive numbers then prove that there is a sequence $\{\epsilon_n\}_{n=1}^{\infty}$ of positive numbers which converges to zero but For which $\sum_{n=1}^{\infty} \epsilon_n a_n$ still diverges. 08

B] Attempt any one:

a) Does the series 07

i) $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$ and

ii) $\sum_{n=1}^{\infty} \frac{n+1}{10^{10}(n+2)}$ converge or diverge?

Justify your answer.

b) Prove that the series $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{n}$ converges. 07

Q.3 A] Attempt any one:

a) If $u_1, u_2, \ldots, u_n$ are implicit functions of $x_1, x_2, \ldots, x_n$ then prove that 05

$$\frac{\partial(u_1, u_2, \ldots, u_n)}{\partial(x_1, x_2, \ldots, x_n)} = (-1)^n \frac{\frac{\partial(F_1, F_2, \ldots, F_n)}{\partial(x_1, x_2, \ldots, x_n)}}{\frac{\partial(F_1, F_2, \ldots, F_n)}{\partial(u_1, u_2, \ldots, u_n)}}$$

b) Prove that the inverse image of the intersection of two sets is the intersection of the inverse images. 05

B] Attempt any one :

c) Find the Jacobian of $y_1, y_2, \ldots, y_n$ being given $y_1 = 1 - x_1, y_2 = x_1(1 - x_2), \ldots, y_n = x_1 x_2 \ldots x_{n-1}(1 - x_n)$. 05

d) If $x = c \cos u \cos hv, y = c \sin u \sin hv$, 05

Prove that $\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{2} c^2 (\cos 2u - \cos h2u)$.

Q.4 Choose correct alternative of the following. 10

1) If $f: A \rightarrow B$ is a function defined by $f(x) = \sqrt{x}$ then
   - a) $A = B = I\mathbb{R}$
   - b) $A = I\mathbb{R}, B = I\mathbb{R}^4$
   - c) $A = I\mathbb{R}^+, B = I\mathbb{R}$
   - d) $A = I\mathbb{R}^+, B = I\mathbb{R}^+$
2) Total number of sequences can be defined whose range set containing either 1 or -1 are ______.
   - a) Countable infinite
   - b) Uncountable infinite
   - c) Two
   - d) One
3) If for every $E > 0$, there exist a positive integer N does not depend on ϵ such that $|S_n - L| < \epsilon$ For all $n \geq N$ then ______
   - a) All but finite number of terms of $\{S_n\}$ are equal to L
   - b) No term of $\{S_n\}$ is equal to L
   - c) Sequence diverges to ∞
   - d) Sequence diverges to $-\infty$
4) The Series $\sum \frac{1}{n}$ is
   - a) Convergent
   - b) Divergent
   - c) Oscillatory
   - d) None of these
5) If $u(x, y) = xy$ and $v(x, y) = x + y$ then Jacobian of u and v is
   - a) x
   - b) y
   - c) $x - y$
   - d) $y - x$