Total No. of Printed Pages: 2

SUBJECT CODE NO:- B-2047
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-V)
Examination November/December- 2022
Mathematics MAT - 502
Abstract Algebra - I

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1 A. Attempt any one of the following: 08

a. If ϕ is a homomorphism of G onto $\bar{G}$ with kernel K then prove that ${}^{G}/_{K} \approx \bar{G}$.

b. If G is a finite group and H is a subgroup of G then prove that order of H is a divisor of order of G.

B. Attempt any one of the following: 07

a. If H is a subgroup of a group G then show that $\{x \epsilon G \mid x h = hx, for\ all\ h \epsilon F\}$ is a subgroup of G.

b. Prove that the subgroup N of a group G is normal subgroup of G if and only if every left coset of N is G is a right coset of N in G.

Q.2 A. Attempt any one of the following: 08

a. If ϕ is a ring homomorphism of R into R then prove that

i) $\phi(0) = 0$

ii) $\phi(-a) = -\phi(a)$, for every $a \in R$.

b. If R is a commutative ring with unit element whose only ideals are {0} and R itself then prove that R is a field.

B. Attempt any one of the following: 07

c. If U is an ideal of a ring R then prove that [R:U]={x$\epsilon R | r.x \epsilon U$ for every $r \epsilon R$ } is an ideal of R.

d. With usual notations prove that F[x] is an integral domain.

Q.3 A. Attempt any one of the following: 05

a. Show that every subgroup of an abelian group is a normal subgroup.

b. If U is an ideal of R and 1 ϵ U then show that U=R

B. Attempt any one of the following: 05

c. Show that $x^3 - 9$ is reducible over the field of integers modulo 11.

d. If G is a group then for all a, b ϵ G prove that $(b.a)^{-1} = a^{-1}.b^{-1}$

Q.4 Choose the correct alternative and rewrite the sentence: 10

1. If o(H) divides o(G) and o(H) $\neq$ o(G) then ________
   a. H is a subgroup of G
   b. G is a subgroup of H
   c. G=H
   d. H may or may not be subgroup of G.

2. If G is the set of all n x n, nonsingular matrices with rational number entries then under matrix multiplication G is
   a. Finite abelian group
   b. Infinite abelian group
   c. Infinite non abelian group
   d. Finite non abelian group

3. The set of all real numbers is not a group under usual multiplication because
   a. The identity does not exist
   b. Multiplication of reals is not associative
   c. Zero has no inverse
   d. Multiplication of reals not satisfy closure property

4. If K is a subgroup of H, H is a subgroup of G and o(K)=2, o(H)=10, o(G)=20 then index of K in G is _____
   a. 2
   b. 10
   c. 20
   d. 40

5. If R is a ring then $(a-b)^2 = ------$
   a. $a^2 - 2ab + b^2$
   b. $a^2 + 2ab + b^2$
   c. $a^2 - ab - ba + b^2$
   d. $a^2 - ab + ba + b^2$