Total No. of Printed Pages: 02

SUBJECT CODE NO:- B-2115
FACULTY OF SCIENCE & TECHNOLOGY
B. Sc. T.Y. (Sem-V)
Examination November/December- 2022
Mathematics
Ordinary Differential Equation -I 504

[Time: 1:30 Hours]   [Max. Marks: 50]

Please check whether you have got the right question paper.

N.B   1) All questions are compulsory
                                       2) Figures to the right indicate full marks

Q.1   A) Attempt any one :

a) Consider the equation $y' + ay = b(x)$ where a is a constant and b is continuous function   08
on an interval I. If x_o is a point in I and C is any constant. Prove that the function ϕ
defined by

$$\phi(x) = e^{-ax} \int_{x_o}^{x} e^{at} b(t) dt + ce^{-ax}$$

is a solution of this equation? Also prove that every solution has this form

b) If r is a root of multiplicity m of a polynomial P. deg $P \geq 1$ then prove that   08
$P(r) = P'(r) = - - - - - - - - = P^{(m-1)}(r) = 0 \ and \ P^{(m)}(r) \neq 0$

B) Attempt any one

c) Find all solutions of the equation $y' + 2xy = xe^{-x^2}$ 07

d) If ϕ be the solution of $y' + iy = x$ such that $\phi(0) = 2$ find $\phi(\pi)$ 07

Q.2   A) Attempt any one

a) Prove that two solutions ϕ_1, ϕ_2 of $L(y) = y'' + a_1 y' + a_2 y = 0$ are linearly independent   08
on an interval I if and only if , $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I

b) If ϕ_1, ϕ_2 are two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a   08
point x_o then prove that
$W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_o)} W(\phi_1, \phi_2)(x_o)$

B) Attempt any one :

c) Find all solutions of the equation $y'' + 4y = cosx$ 07

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d) Find the solution of the following initial value problem   07
$$y'' + (4i + 1)y' + y = 0 , \qquad y(0) = 0, y'(0) = 0$$

Q.3   A) Attempt any one:

a) Prove that for all real θ 05
$$e^{i\theta} = cos\theta + i\ sin\theta$$

b) Prove that for any real x_o and constants $\propto, \beta$ there exists a solution ϕ of the initial value   05
problem
$$L(y) = y'' + a_1 y' + a_2 y = 0$$
on $-\infty < x < \infty$

B) Attempt any one

c) Find the two square roots of i .   05

d) Find all solutions ϕ of $y'' + y = 0$ 05
$$\phi(0) = 0, \phi'(\pi/2) = 0$$

Q.4   Choose the correct alternative   10

1) The wronskian of the functions
$\phi_1(x) = \sin x$, $\phi_2(x) = e^{ix}$ is
a) 0   b) 1   c) -1   d) None of these

2) The roots of the equation $Z^2 + Z - 6 = 0$ are
a) -3,2   b) 2,3   c) 3,-2   d) none of these

3) If $\phi(x) = e^{iax}$ where a is a real constant then $\phi''(x) + a^2\phi(x) =$-------
a) 1   b) 0   c) e^{iax} d) none of these

4) $\phi(x) = e^{-sinx}$ is a solution of the differential equation?
a) $y' + (cosx)y = 0$ b) $y' - (cosx)y = 0$ c) $y' + (sinx)y = 0$ d) None of these

5) All solutions of $y'' + \omega^2 y = 0$ are of the to form
a) $Ge^{i\omega x} - c_2 e^{-i\omega x}$ b) $Ge^{\omega x} + c_2 e^{-\omega x}$ c) $Ge^{i\omega x} + c_2 e^{-i\omega x}$ d) none of these

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