Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2061
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-VI)
Examination November/December- 2022
Mathematics MAT-601
Real Analysis-II

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B i) All questions are compulsory.
ii) Figures to the right indicate full marks.

Q.1 A. Attempt any one: 08

a) Let $\langle M_1 , P_1\rangle$ and $\langle M_2 , P_2\rangle$ be metric space and let $f : M_1 \to M_2$. Then prove that f is continuous on M_1 if and only $f^{-1}(G)$ is open in M_1 whenever G is open in M_2.

b) If E is any subset of a metric space M , then prove that $\bar{E}$ is closed.

B. Attempt any one: 07

c) Show that if $\rho\ and\ \sigma$ are both metrics for a set M , then $\rho + \sigma$ is also a metric for M.

d) $If\ f : R^2 \to R^2\ is\ defined\ by\ f(\langle x , y\rangle) = (\langle y , x\rangle) \qquad (\langle x , y\rangle) \epsilon\ R^2,$
$show\ that\ f\ is\ continuous\ on\ R^2.$

Q.2 A. Attempt any one: 08

a) Prove that the metric space $\langle M , P\rangle$ is compact if and only if every sequence of points in M has a subsequence converging to a point in M.

b) Let f(x) be Riemann integrable in every interval and is periodic with 2π as its period , then prove that $\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} f(a + x)dx$ where a is any number.

B. Attempt any one: 07

c) Prove that R^2 is complete.

d) For each
n ϵ I let $b_n$be the subdivision $\{0 , {}^{1}/_{n} , {}^{2}/_{n} , \dots , {}^{n}/_{n}\}\ of\ [0 , 1]. Compute\ \lim_{n\to\infty} L[f ; \sigma n]$
for the function $f(x) = x^2(0 \le x \le 1)$.

Q.3 A. Attempt any one: 05

a) Let f be a continuous function from the compact metric space M_1 into the metric space M_2. Then prove that the range $f(M_1)$ of f is also compact.

b) If f is a continuous function on the closed bounded interval [a , b] , and if $\Phi'(x) = f(x)$ $(a \le x \le b)$ then prove that $\int_a^b f(x)dx = \Phi(b) - \Phi(a)$.

B. Attempt any one: 05

c) Find the Fourier series of f(x) = x in [-π, π].

d) $If\ 0 \le x \le 1\ show\ that\ \frac{x^2}{\sqrt{2}} \le \frac{x^2}{\sqrt{1+x}} \le x^2$.

Q.4 Choose the correct alternative: 10

I) The convergent sequence in a metric space has -----.
a) Unique limit c) Limit ∞
b) Distinct limit d) None of these

II) If $\langle M , P\rangle = R^1\ and\ \langle A , P\rangle = [0 , 1]$, then the open ball $B\left[0 ; \frac{1}{2}\right]\ in\ R^1$ is the interval -----.
a) $[-\frac{1}{2} , \frac{1}{2}]$ c) $(-\frac{1}{2} , \frac{1}{2}]$
b) $(0 , \frac{1}{2})$ d) $(-\frac{1}{2} , \frac{1}{2})$

III) The metric space [a, b] with absolute-value metric is -------.
a) Only totally bounded c) Bounded
b) Only complete d) Totally bounded and complete

IV) If f is a bounded function on the closed bounded interval [a , b] and σ is any subdivision of [a , b] , then $\int_{-a}^{b} f(x)dx = ----$.
a) $l . u . b . \cup [f , \sigma]$ c) $l . u . b . L [f ; \sigma]$
b) $g . l . b . \cup [f ; \sigma]$ d) $g . l . b . L[f , \sigma]$

V) For all n = 0 , 1 , 2 , …….., $\int_{-\pi}^{\pi} \cos^2 nx\ dx = ------$.
a) 0 b) π c) π/4 d) π^2