Total No. of Printed Pages:3

SUBJECT CODE NO:- B-2123
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-VI)
Examination November/December- 2022
Mathematics
Ordinary Differential Equation-II - MAT- 604

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory
2) Figures to the right indicate full marks

Q.1 A) Attempt any one: 08

a) Let $\phi_1, \dots, \phi_n$ be n linearly independent solutions of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = 0$ on an interval I. prove that if ϕ is any solution of L(y) =0 on I it can be represented in the form $\phi = c_1\phi_1 + ------ + c_n\phi_n$ where $c_1, \dots, c_n$ are constants

b) Let ϕ_1 be a solution of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = 0$ on an interval I and suppose $\phi_1(x) \neq 0$ on I. if $v_2, v_3, \dots, v_n$ *is any basis* on I for the solutions of the linear equation.

$\phi_1\vartheta^{(n-1)} + \cdots + \left[n\phi_1^{(n-1)} + (n-1)\phi_1^{(n-2)} + \cdots + a_{n-1}\phi_1\right]\vartheta = 0$

of order n-1 , and if

$\vartheta_k = u_k', \quad (k = 2, \dots, n)$

Then prove that $\phi_1, u_2\phi_1, \dots, u_n\phi_1$ is a basis for the solutions of $L(y) = 0$ *on* I.

B) Attempt any one 07

c) Find two linearly independent solutions of the equation $(3x-1)^2y'' + (9x-3)y' - 9y = 0$ for $x > \frac{1}{3}$

d) Find all solutions of $y'' - \frac{2}{x^2}y = 0, (0 < x < \infty)$ given that one solution is $\phi_1(x) = x^2$

Q.2 A) Attempt any one 08

a) If $\phi_1, \dots, \phi_n$ are n solutions of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = 0$ on an interval I, prove that they are linearly independent if and if $W(\phi_1, \dots, \phi_n)(x) \not\supset 0$ for all x in I

b) If ϕ_1 is a solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on I and $\phi_1(x) \neq 0$ on I prove that a second solution $\phi_2(x)$ of L(y) =0 is given by

$$\phi_2(x) = \phi_1(x) \int_{x_o}^{x} \frac{1}{[\phi_1(s)]^2} \exp[-\int_{x_o}^{s} a_1(t)dt\,]\,ds$$

B) Attempt any one 07

c) Find two linearly independent power series solutions of the equation

$$y'' + 3x^2y' - xy = 0$$

d) Show that

$$\int_{-1}^{1} P_n(x)P_m(x)dx = 0\,, (n \neq m)$$

Q.3 A) Attempt any one 05

a) One solution of $xy'' - (x+1)y' + y = 0, (x > 0)$ is given as $\phi_1(x) = e^x$ find the second solution

b) Find all solutions of the equation $2x^2\, y'' + xy' - y = 0 \qquad (x > 0)$

B) Attempt any one 05

c) Compute the indicial polynomial and their roots for the equation $x^2y'' + (x + x^2)y' - y = 0$

d) Find all solutions ϕ of the form $\phi(x) = |x|^r \sum_{k=0}^{\infty} c_k x^k \qquad (|x| > 0)$
For the equations $x^2y'' + xy' + x^2y = 0$

Q.4 Choose the correct alternative 10

1) One solution of $x^2y'' - xy' + y = 0 \quad (x > 0)$ is
a) $\phi(x) = x$ b) $\phi(x) = x^2$ c) $\phi(x) = e^x$ d) $\phi(x) = e^{-x}$

2) The Bessel equation is of the form
a) $(1 - x^2)y'' - 2xy' + \propto (\propto +1)y = 0 \;\; \propto is\ constant$
b) $x^2y'' + xy' + (x^2 - \propto^2)y = 0\,,\;\; Re \propto \geq 0$
c) $x^2y'' + axy' + by = 0$, a,b constant
d) $x^2\, y'' + 5y' + 3x^2y = 0$

3) The indicial polynomial of the equation $L(y) = x^2y'' + axy' + by = 0$ a, b constants is
a) $q(r) = r(r+1) + ar + b$
b) $q(r) = r(r-1) - ar + b$
c) $q(r) = r(r-1) + ar - b$
d) $q(r) = r(r-1) + ar + b$

4) The solutions of the equation $x^2y'' + 2xy' - 6y = 0$ for $x > 0$ are :
   a) x^2, x^3 b) x^{-2}, x^3 c) x^{-3}, x^2 d) x^{-2}, x^{-3}

5) The n-th degree Legendre polynomial $P_n(x) is\ given\ by$
   a) $\frac{1}{2^n n!}\ \frac{d^n}{dx^n}\ (x^2+1)^n$

   b) $\frac{1}{2^n n!}\ \frac{d^n}{dx^n}\ (x^2-1)^n$

   c) $\frac{1}{2^n n!}\ \frac{d^n}{dx^n} x^{2n}$

   d) $\frac{(2n)!\ \ x^{2n}}{2^n(n!)^2}$