Total No. of Printed Pages:3

SUBJECT CODE NO:-B-2062
FACULTY OF SCIENCE & TECHNOLOGY
B.Sc. T.Y. (Sem-VI)
Examination November/December- 2022
Mathematics MAT - 602
Abstract Algebra - II

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
i) All questions are compulsory
ii) Figures to the right indicate full marks.

Q.1 (A) Attempt any one: 08

(a) Prove that if U is a vector space over F and W is a subspace of U, then there is a homomorphism of U onto U/W.

(b) If V is finite-dimensional and if W is a subspace of V, then prove that W is finite-dimensional and dim W $\leq$ dim V.

(B) Attempt any one: 07

(c) Prove that the intersection of two subspaces of a vector space V is a subspace of V.

(d) If W_1 and W_2 are subspaces of finite-dimensional vector space V over F, then show that

$$A(W_1 + W_2) = A(W_1) \cap A(W_2).$$

Q.2 (A) Attempt any one: 08

(a) Prove that a homomorphism T of an R-module M into an R-module N with kernel K(T) is an isomorphism if and only if K(T)=(O).

(b) If V is a finite-dimensional inner product space and if W is a subspace of V, then prove that $V=W+W^{\perp}$.

(B) Attempt any one: 07

(c) If S is subset of a vector space V, let $A(S) = \{f \in \hat{V} | f(s) = 0 \text{ for all } s \in S\}$. Prove that A(S)=A(L(S)), where L(S) is the linear span of S.

(d) If F is the real field and V is $F^{(3)}$, show that the Schwarz inequality implies that the cosine of an angle is of absolute value at most one.

Q.3 (A) Attempt any one: 05

(a) If S is nonempty subset of the vector space V, then prove that L(S) is a subspace of V.

(b) If u, v ∈ V and $\alpha, \beta \in F$, then prove that $\|\alpha u + \beta v\|^2 = |\alpha|^2\|u\|^2 + \alpha\bar{\beta}(u, v) + \bar{\alpha}\beta(v, u) + |\beta|^2\|v\|^2$.

Where V is an inner product space over F.

05

(B) Attempt any one:

(c) Show that in $F^{(3)}$ the vectors (1,0,0), (0,1,0) (0,0,1) are linearly independent.

(d) If V is finite-dimensional and $V_1 \neq V_2$ are in V, prove that there is an $f \in \hat{V}$ such that $f(V_1) \neq f(V_2)$.

Q.4 Choose the correct alternative: 10

(i) If V is vector space over a field F, then the subspace V itself and (O) of V are called ________.

(a) Proper subspaces (b) Improper subspaces

(c) Modules (d) None of these

(ii) If W is subspace of a vector space V over F such that dim V=8 and dim W=5, then dim A(W) ________.

(a) 13 (b) 8

(c) 3 (d) 5

(iii) The number of elements in two basis of a finite dimensional vector space is ------

(a) Equal (b) Unequal

(c) May or may not be equal (d) None of these

(iv) The dimension of a vector space R^3 over R is ________.

(a) 2 (b) 4

(c) 1 (d) 3

(v) An orthogonal set of non-zero vectors is ________.

(a) Linearly dependent

(b) Linearly independent

(c) A basis

(d) None of these