Total No. of Printed Pages:02

SUBJECT CODE NO:- 2039
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y Sem. I
Examination March/April-2022 (To Be Held In June/July-2022)
Mathematics MAT - 101
Differential Calculus

[Time: 1:53 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1. Attempt all questions.
2. Figures to the right indicate full marks.

Q.1 A. Attempt any one. 08

a) If f is finitely derivable at C. then prove that f is continuous at C.

b) Prove that $\frac{d^n}{dx^n}\left(\frac{1}{ax+b}\right) = \frac{(-1)^n\, n! a^n}{(ax+b)^{n+1}}$

07

B. Attempt any one.

c) Find $\frac{dy}{dx}, if\ y = \frac{x^3\sqrt{x^2+4}}{\sqrt{x^2+3}}$

d) If $cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, Prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$

Q.2 A. Attempt any one 08

a) If a function f is
   i) Continuous in a closed interval [a, b] and
   ii) Derivable in open interval (a, b) then there exists at least one value $c \in (a, b)$ such that
   $$\frac{f(b)-f(a)}{b-a} = f'(c)$$

b) If $z = f(x, y)$ is a homogeneous function of x, y of degree n, then prove that
$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$$

07

B. Attempt any one.

c) Verify Cauchy Mean Value Theorem for the functions x^2 and x^4 in the interval [a, b]; a, b being positive.

d) If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}; xy \neq 0$, Prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}$

Q.3 A) Attempt any one 05

a) Prove that curl $(\emptyset\vec{f})$=grad $\Phi \times \vec{f} + \Phi\, curl\, \vec{f}$

b) Prove that div $\vec{f}$ is a point function.

B) Attempt any one 05

c) If $\vec{f} = x^2 z\,\vec{i} - 2\,y^3z^2\vec{j} + xy^2z\vec{k}$, then find div $\vec{f}$ and curl $\vec{f}$ at (1,-1,1)

d) Find grad $\Phi, if\ \Phi = 3y - y^3z^2\ at\ the\ point\ (1,-2,-1)$

Q.4 Choose the correct alternative. 10

i) If $f(x) = |x|, then\ f$ if not derivable at _____

a. 1 b. 0 c. -1 d. 2

ii) If $x = a(1-\cos\theta), y = a(\theta - sin\theta)$ then $\left(\frac{dy}{dx}\right)_{\theta=\pi/2}$

a. -2 b. -4 c. 1 d. -1

iii) If $z = f(x,y)$ be a homogeneous function of x, y of degree n. then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ = ___.

a. nz b. n(n-1)z

c. n(n+1)z d. none of these

iv) A polynomial function in R _____

a. Is never continuous in R

b. May or may not be continuous in R

c. Is always continuous in R

d. Is continuous for all values of x except finitely many.

v) Div $\vec{r}$ = _____

a. 1 b. 2 c. 3 3. 0