Total No. of Printed Pages: 2

SUBJECT CODE NO:- 2051
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y (Sem. III)
Examination March/April-2022 (To Be Held In June/July-2022)
Mathematics MAT - 302
Integral Transforms

[Time: 1:53 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B.
i) All questions are compulsory.
ii) Figures to the right indicate full marks.

Q.1 (a) Attempt any one of the following : 08

i. If a function $F(t)$ is periodic function with period ω, so that $F(t + n\omega) = F(t)$ for $n = 1, 2, ...$, prove that

$$L\{F(t)\} = \frac{1}{1 - e^{-s\omega}} \int_0^\omega e^{-su} F(u)du.$$

ii. If $l, m > 0$, then prove that $B(l,m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$.

(b) Attempt any one of the following: 07

i. Using Heavi-side's expansion formula find $L^{-1}\left\{\frac{2s^2+5s-4}{s^3+s^2-2s}\right\}$.

ii. Using Laplace transform, find the solution of the differential equation

$$y'' + 25y = 10 \cos 5t,$$

where $y(0) = 2$ and $y'(0) = 0$.

Q.2 (a) Attempt any one of the following: 08

i. If $f(s)$ is the Fourier transform of $F(x)$, then show that the Fourier transform of $F'(x)$ is equal to $i\, s\, f(s)$.

ii. If $L\{F_1(t)\} = f_1(s)$ and $L\{F_2(t)\} = f_2(s)$, then prove that
$L^{-1}\{c_1 f_1(s) + c_2 f_2(s)\} = c_1 L^{-1}\{f_1(s)\} + c_2 L^{-1}\{f_2(s)\}$
where c_1 and c_2 are any constants.

(b) Attempt any one of the following: 07

i. Find the Fourier transform of

$$f(x) = \begin{cases} 1, & \text{if } |x| < a, \\ 0, & \text{if } |x| > a. \end{cases}$$

ii. Prove that $L\{(1 + te^{-t})^3\} = \frac{1}{s} + \frac{3}{(s+1)^2} + \frac{6}{(s+2)^3} + \frac{6}{(s+3)^4}$.

Q.3 (a) Attempt any one of the following: 05

i. If $L\{F(t)\} = f(s)$, then prove that $L\{e^{at} F(t)\} = f(s - a)$.

ii. If $f(s)$ is the Fourier transform of $F(x)$, then prove that $\frac{1}{a} f\left(\frac{s}{a}\right)$ is the Fourier transform of $F(ax)$.

(b) Attempt any one of the following: 05

i. Prove that $L\{e^{2t}(cos\ 4t + \ 3\ sin\ 4t)\} \ = \frac{s\ +\ 10}{s^2-4s+20}$.

ii. Show that x^n is of exponential order as $x \to \pm\infty$, n being any integer.

Q.4 Choose the correct alternative and rewrite the sentence : 10

(a) If n is a positive integer, then $\Gamma(n)$ = ______.

i. n

ii. $(n - 1)$

iii. $(n - 1)!$

iv. $n!$

(b) If $\int_0^\infty \left(\frac{e^{-at}-e^{-bt}}{t}\right) dt = log\frac{b}{a}$, then $\int_0^\infty \left(\frac{e^{-t}-e^{-5t}}{t}\right) dt =$ ______

i. log 4

ii. log 5

iii. log 6

iv. log 2

(c) The Fourier sine transform of $f(x) \ = \frac{1}{x}$ is ______

i. π

ii. $\frac{\pi}{2}$

iii. $\frac{\pi}{3}$

iv. $\frac{\pi}{4}$

(d) $L^{-1}\left\{\frac{s}{s^2+a^2}\right\} =$ ______

i. sin t

ii. sin at

iii. cos t

iv. cos at

(e) If $L\{F(t)\} = f(\text{s})$, then $L\{F'(t)\}$ = ______.

i. $sf(s) - F(0)$

ii. $sf(s) - F(s)$

iii. $sf(s) + F(0)$

iv. $sf(s) - sF(0)$