Total No. of Printed Pages:2

SUBJECT CODE NO:- 2050
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y (Sem-III)
Examination March/April-2022 (To be held in June/July-2022)
Mathematics MAT – 301
Number Theory

[Time: 1:53 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B i) Attempt all questions.
ii) Figures to the right indicate full marks.

Q. 1 (a) Attempt **any one** of the following: 08

i. For positive integers a and b prove that $Gcd(a,b)\ lcm(a,b) = ab$.

ii. If a and be are integers, not both zero, then prove that a and be are relatively prime if and only if there exist integers $x\ and\ y\ such\ that\ 1 = ax + by$.

(b) Attempt **any one** of the following: 07

i. By using the Euclidean algorithm, find the values of integers $x\ and\ y\ satisfying\ ged\ (24,138) = 24x + 138y$.

ii. Solve the linear Diophantine equation 172x + 20y = 1000.

Q. 2 (a) Attempt **any one** of the following 08

i. Prove that the linear congruence $ax \equiv b(\mathrm{mod}n)$ has a solution if and only if d/b, where d = gcd(a, n). If d/b then it has d mutually incongruent solutions modulo n.

ii. State and prove Wilson's theorem.

(b) Attempt **any one** of the following: 07

i. Use Fermat's theorem to verify that 17 divides $11^{104} + 1$.

ii. Solve the following set of simultaneous congruences $x \equiv 5(mod6), x \equiv 4(mod11), x \equiv 3(mod17)$.

Q. 3 (a) Attempt **any one** of the following: 05

i. Show that $\sqrt{2}$ is irrational number:

ii. If f is multiplicative function and F is defined by $F(n) = \sum_{d/n} f(d)$,
Then show that F is also multiplicative function.

(b) Attempt **any one** of the following: 05

i. Calculate $\phi(5040)$.

ii. Find the remainder when 2^{50} is divided by 7.

Q. 4 Choose the correct alternative and **rewrite the sentence**: 10

(a) The square of any odd integer is of the form ____

i. $5k + 1$
ii. $8k + 1$
iii. $3k + 1$
iv. $8k$

(b) Lcm(-12, 30) = ____

i. 30
ii. –12
iii. 60
iv. 360

(c) Number of solutions for the linear congruence $9x \equiv 21(mod\ 30)$ are _____

i. 6
ii. 1
iii. 4
iv. 3

(d) The value of $\sigma(20) =$ _____

i. 28
ii. 12
iii. 42
iv. 54

(e) If p is a prime, and $k > 0, then\ \phi\ (p^k) =$ _____

i. $k - 1$
ii. pk
iii. $p^{k-1} - p^k$
iv. $p^k - p^{k-1}$