Total No. of Printed Pages:2

SUBJECT CODE NO:- 2046
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem. V)
Examination March/April-2022 (To Be Held In June/July-2022)
Mathematics MAT - 501
Real Analysis – I

[Time: 1:53 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1) all questions are compulsory
2) figures to the right indicate full marks

Q.1 A) Attempt any one

1) Prove that limit of a sequence $\{S_n\}_{n=1}^{\infty}$ of nonnegative numbers is also nonnegative number 08
2) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent then prove that $\{S_n\}_{n=1}^{\infty}$ is bounded 08

B) Attempt any one

a) Find the limit superior and limit inferior for the sequence $\left\{\sin \frac{n\pi}{2}\right\}_{n=1}^{\infty}$ 07
b) If $S_n = \frac{10^n}{n!}$ find $N \epsilon I$ such that $S_{n+1} < S_n \quad (n \geq N)$ 07

Q.2 A) Attempt any one

a) If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B then prove that $\sum_{n=1}^{\infty}(a_n + b_n)$converges to CA 08

b) Prove that i) If $\sum_{n=1}^{\infty} a_n$ converges absolutely then both $\sum_{n=1}^{\infty} p_n$ & $\sum_{n=1}^{\infty} q_n$ converges 08
ii) If $\sum_{n=1}^{\infty} a_n$ converges conditionally then both $\sum_{n=1}^{\infty} p_n$ & $\sum_{n=1}^{\infty} q_n$ diverges where $p_n = \max(a_n, o)$& $q_n = \min(a_n, o)$

B) Attempt any one

c) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges 07
d) If $\sum_{n=1}^{\infty} a_n$ is a convergent series then prove that $\lim_{n\to\infty} a_n = o$ 07

Q.3 A) Attempt any one

a) Prove that inverse image of the union of two sets is the union of the inverse images 05
b) If $u_1, u_2, -----, u_n$ are functions of $y_1, y_2, ------, y_n$ and $y_1, y_2, ----, y_n$ are functions of $x_1, x_2, ----, x_n$then prove that 05

$$\frac{\partial(u_1, u_2, ---, u_n)}{\partial(x_1, x_2, ----, x_n)} = \frac{\partial(u_1, u_2, ----, u_n)}{\partial(y_1, y_2, -----y_n)} \cdot \frac{\partial(y_1, y_2, ---, y_n)}{\partial(x_1, x_2, ----, x_n)}$$

B) Attempt any one

c) If $x = r\, sin\theta \cos\phi, \quad y = r \sin\theta \sin\phi$, 05

$$Z = r\cos\theta \;\; Find \;\; \frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$$

d) Prove that the set of all polynomial functions with integer coefficient is countable 05

Q.4 Choose correct alternative of the following 10

1) If $F(x) = [x], where\ [.]$ is the greatest integer function on IR to IR then which of the following is true
   a) f(x) is one –one b) f(x) is onto
   c) f(x) is both one –one and onto d) f(x) is neither one –one nor onto

2) If $S_n = n^2$ then the sequence whose n^{th} term $\frac{1}{S_n}$ is -------
   a) Converges to zero b) converges to one c) oscillatory d) divergent

3) If $p(x) = a_1x^3 + b_1x^2 + c_1x + d_1$ where a,b,c,d, ϵIR^+ then $\lim_{n\to\infty} \frac{p(n+1)}{p(n)} = ----$
   a) 0 b) 1 c) depends on a_1, b_1, c_1 & d_1 d) does not exist

4) A series $\sum a_n$ is said to be absolutely convergent if
   a) $\lim_{n\to\infty} a_n = o$ b) $\lim_{n\to\infty} a_n = +ve$ c) $\sum a_n$ is divergent d) $\sum |a_n|$ is convergent

5) If u(x,y) =x+y and v(x,y) =xy then Jacobian of u and v is
   a) x b) y c) x-y d) y-x