Total No. of Printed Pages: 3

SUBJECT CODE NO:- 2115
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y Sem. V
Examination March/April-2022 (To Be Held In June/July-2022)
2) Ordinary Differential Equation -I 504 OR

[Time: 1:53 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B.
i) All questions are Compulsory.
ii) Figures to the right indicate full marks.

Q.1 (A) Attempt any one :

a) Consider the equation $y' + ay = 0$. Where a is a Complex Constant. If C is any Complex number, Prove that the function ϕ defined by $\phi(x) = ce^{-ax}$ is a Solution of this equation. 08

b) If the determinant Δ of the Coefficients in 08

$$a_{11}z_1 + a_{12}z_2 + \cdots + a_{1n}z_n = c_1$$
$$a_{21}z_1 + a_{22}z_2 + \cdots + a_{2n}z_n = c_2$$
$$\vdots$$
$$a_{n1}z_1 + a_{n2}z_2 + \cdots + a_{nn}z_n = c_n$$

is not zero. Prove that there is a unique Solution of the System for $z_1, z_2, \ldots\ldots, z_n$. It is given by $z_K = \frac{\Delta_K}{\Delta}$, (K = 1, 2, 3, …, n)

Where Δ_K is the determinant obtained from Δ by replacing its Kth Column $a_{1K}, \ldots, a_{nK}$ by $C_1, \ldots, C_n$.

(B) Attempt any one :

c) Find all solutions of the equation $y' - 2y = x^2 + x$. 07

d) Consider the equation $y' + (\cos x)y = e^{-\sin x}$. 07

a) Find the solution ϕ which Satisfies $\phi(\pi) = \pi$

b) Show that any solution ϕ has the property that $\phi(\pi K) - \phi(0) = \pi K$.
Where K is any integer.

Q.2 (A) Attempt any one :

a) State and Prove Uniqueness theorem for the equation $L(y) = y'' + a_1y' + a_2y = 0$. 08

b) If ϕ_1, ϕ_2 be two Solutions of $L(y) = 0$ on an interval I, and x_0 be any point in I. Then Prove that ϕ_1, ϕ_2 are linearly independent on I if and only if
$W(\phi_1, \phi_2)(x_0) \neq 0$.
where $L(y) = y'' + a_1y' + a_2y$. 08

(B) Attempt any one :

c) Find the solution of the following Initial Value Problem. 07
$y'' + 10y = 0$, $y(0) = \pi$, $y'(0) = \pi^2$.

d) Find all Solutions of 07

$y'' + 9y = \sin 3x$

Q.3 (A) Attempt any one :

a) If P be a Polynomial of degree $n \geq 1$, with leading Coefficient one and r be a root of P. Then Prove that
$P(z) = (z - r)\, q(z)$.
Where q is a Polynomial of degree n-1 with leading coefficient one. 05

b) If for all real x
$f(x) = x + ix^2$, $g(x) = \frac{x^2}{2}$
Compute: i) The function F given by $F(x) = f(g(x))$.
ii) $F'(x)$. 05

(B) Attempt any one :

c) Consider the equation $y' + 5y = 2$. 05

i) Show that the function ϕ given by
$\phi(x) = \frac{2}{5} + ce^{-5x}$
is a Solution, where c is any constant.

ii) Find that solution satisfying $\phi(1) = 2$

d) Determine whether the functions ϕ_1, ϕ_2 defined below are linearly dependent or independent. 05
$\phi_1(x) = \cos x$, $\phi_2(x) = \sin x$.

Q.4 Choose the Correct alternative. 10

i) The Wronskian of the functions $\phi_1(x) = x^2$, $\phi_2(x) = 5x^2$
a) 0
b) 1
c) -1
d) None of these.

ii) The roots of the characteristic polynomial $r^2 - r - 2 = 0$ are
a) –1, –2
b) –1, 2
c) 1, –2
d) None of these.

iii) The Solutions of $y' = Ky$ are given by
a) ce^{Kx}
b) ce^{-Kx}
c) ce^{x}
d) None of these.

iv) If a, b, x are real then $Re(e^{(a+2b)x})$ is
a) e^{ax}
b) $e^{ax}\cos bx$
c) $e^{ax}\sin bx$
d) None of these.

v) The equation $y' + a(x)y = 0$ is called _____.

a) Homogeneous equation
b) Non-homogeneous equation
c) Both (a) and (b)
d) None of these.