Total No. of Printed Pages:2

[Time: 1:53 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i) All questions are compulsory.
ii) Figures to the right indicate full mark.

Q.1 A) Attempt any one of the following:

    a) Define subgroup of group G. If H is a subset of G such that $a \cdot \bar{b}^1 \in H$ for all a, b, $\in$ H then prove that H is a subgroup of G.   8

    b) If ϕ is homomorphism of G into $\bar{G}$ with kernel K_ϕ then prove that ϕ is an isomorphism if and only if $K_\phi = \{e\}$, where e is the identity in G.   8

B) Attempt any one of the following:

    c) If G in a group and $a \in G$ then show that $\{N(a) = |x \, a = ax\}$ is a subgroup of G.   7

    d) Is intersection of two normal subgroups of a group G again a normal subgroup? Justify your answer.   7

Q.2 A) Attempt any one of the following:

    a) If the number of elements in the integral domain D are finite, then prove that D is a field.   8

    b) If $f(x), g(x) \in F[x]$ such that $p(x) = f(x) \cdot g(x) \neq 0$ and $\deg(f(x)) = m$, $\deg(g(x)) = n$ then prove that $\deg(p(x)) = m + n$.   8

B) Attempt any one of the following:

    c) If R is the ring of all the real valued, continuous functions on the closed unit interval then show that $M = \{f(x) \in R \mid f\left(\frac{1}{3}\right) = 0\}$ is a maximal ideal of R.   7

    d) If U is an ideal of ring R then show the $r(U) = \{x \in R \mid xu = 0 \text{ for all } u \in U\}$ is also an ideal of R.   7

Q.3 A) Attempt any one of the following:

    a) If G is a group such that $(a \cdot b)^2 = a^2 \cdot b^2$ for all a, b, $\in$ G then show that G in an abelian group.   5

    b) In R is a ring, define $\phi: R \to R$ by $\phi(x) = x$ then find kernel of ϕ and hence show that ϕ in an isomorphism.   5

B) Attempt any one of the following:

    c) Prove that any field has improper ideals.   5   5

d) If G is the group of all real numbers under addition and H is the set of all integers then show H is a subgroup of G.

Q.4 Choose the correct alternative and rewrite the sentence: 10

1) Which of the following in not a field
   (a) Z_5
   (b) Z_{13}
   (c) Z_{21}
   (d) Z_{23}

2) If * is the binary operation defied on $\mathbb{R} \times \mathbb{R}$ by $(a, b) * (c, d) = (ad + bc, bd)$ and * satisfies associative law then $(1, 2) * (3, 5) * (3, 4) = $ _____
   (a) (74, 40)
   (b) (32, 40)
   (c) (23, 11)
   (d) (7, 11)

3) The characteristic of an integral domain is either zero or _____
   (a) One
   (b) Any real number
   (c) Prime number
   (d) Composite number

4) The set of all cube roots of unity under multiplication of complex numbers in _____
   (a) Not a group
   (b) Infinite group
   (c) Finite abelian group
   (d) Infinite abelian group

5) If R is a ring with unit element 1 and U is an ideal of R then the unit element in quotient ring R/U is unit element in quotient ring R/U is
   (a) U
   (b) U + R
   (c) 1
   (d) 1 + U

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