Total No. of Printed Pages:03

SUBJECT CODE NO:- 2123
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y Sem-VI
Examination March/April-2022 (To be held in June/July-2022)
Mathematics
Ordinary Differential Equation-II - MAT- 604

[Time: 1:53 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1 A) Attempt any one : 08

a) Prove that there exist n linearly solutions of _
$$L(y) = y^{(n)} + a_1 + (x)y^{(n-1)} + \ldots + an(x)y = 0 \; on \; I.$$

b) If $\phi_1, \phi_2, \ldots, \phi_n$ are n solutions of 08
$L(y) = y^{(n)} + a_1 + (x)y^{(n-1)} + \ldots + an(x)y = 0 \; on$ an interval I, prove that they are linearly independent there if and only if $W(\phi_1, \phi_2, \ldots, \phi_n)(x) \neq 0$ for all x in I.

B) Attempt any one:

07

c) One solution of
$x^3y''' - 3x^2y'' + 6xy' - 6y = 0, for \; x > 0 \; is \; \phi_1(x) = x.$ find the basis for the solutions for $x > 0$

07

d) One solution of
$x^2y'' - 2y = 0 \; on \; 0 < x < \infty \; is \; \phi_1(x) = x^2.$ find all solutions of
$x^2y'' - 2y = 2x - 1 \; on \; 0 < x < \infty.$

Q.2 A) Attempt any one: 08

a) Suppose that $\phi_1, \phi_2, \ldots, \phi_n$ are n solutions of
$L(y) = y^{(n)} + a_1 + (x)y^{(n-1)} + \ldots + an(x)y = 0 \; on$ I Satisfying

$\phi_i^{(i-1)}(xo) = 1, \phi_i^{(i-1)}(xo) = 1. j \neq i$
If ϕ is any solution of $L(y) = 0 \; on \; I$, Prove that there are n constants $c_1, c_2, \ldots, c_n$ such that.
$\phi = c_1\phi_1 + c_2\phi_2 + \ldots + c_n\phi_n$

08

b) Consider the second order Euler equation
$x^2y'' + axy' + by = 0$ (a, b constants) and the polynomial q is given by
$q(r) = r(r-1) + ar + b$
Prove that a basis for the solutions of the Euler equation on any interval not containing $x = 0$

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Is given by
$\phi_1(x) = |r|^{r_1}, \phi_2(x) = |x|^{r_2}$
In case r_1,r_2 are distinct roots of q.

B) Attempt any one. 07

c) Find two linearly independent solutions of the equation
$(3x-1)^2\, y'' + \; (9x-3)y' - 9y = 0 \; for \; x > 1/3$
07

d) Find two linearly independent power series solutions of the equation.
$y'' - x^2 y = 0$

Q.3 A) Attempt any one : 05

a) Show that :
$$\int_{-1}^{1} Pn^2(x)dx = \frac{2}{2n+1}$$
05

b) Show that the coefficient of x^nin
$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n$ is $\frac{(2n)!}{2^n (n!)^2}$

B) Attempt any one : 05

c) Find all solutions of the equation
$x^2y'' + xy' + 4y = 1, x| > 0$ 05

d) Suppose that ϕ is any solution of
$x^2y'' + xy' + x^2y = 0 \; for \; x > 0 \; and \; \psi(x) = x^{1/2} \; \phi(x).$
$show \; that \; \psi \; satisfies \; the \; equation$
$$x^2y'' + \left(x^2 + \frac{1}{4}\right)y = 0, for \; x > 0$$

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Q.4 Choose the correct alternatives: 10

i) If $\phi_1, \phi_2, \dots, \phi_n$ are n solutions of $y^{(n)} + a_1(x)y^{(n-1)} + \dots + an(x)y = 0$ on an interval I and x_o be any point in I. Then ___________

a. $W(\phi_1, \phi_2, \dots, \phi_n)\ (x) = \exp[\int_{x0}^{x} a_1(t)dt$

b. $W(\phi_1, \phi_2, \dots, \phi_n)\ (x) = \exp[-\int_{x0}^{x} a_1(t)dt]W\ (\phi_1, \phi_2 \dots \phi_n)(x_o)$

c. $W(\phi_1, \phi_2, \dots, \phi_n)\ (x) = \int_{x_0}^{x} a_1(t)dt$

d. $W(\phi_1, \phi_2, \dots, \phi_n)\ (x) = \exp[a_1(t)dt]W(\phi_1, \phi_2, \dots, \phi_n)(x_0)$

ii) The function $J_o(x) = \sum_{m=o}^{\infty} \frac{(-1)^m}{m=o(m!)} \left(\frac{x}{2}\right)^{2m}$ is solution of _________

a. Legendre equation
b. Bessel equation
c. Euler's equation
d. 'first order linear equation

iii) If $P_n(-x) = (-1)^n\ P_n\ (x), then\ P_n(-1) = \cdots$

a) $(-1)^n$

b) $(1)^n$

c) 1

d) n

iv) The solution of the equation $x^2y'' + xy' + y = 0\ for\ x > o\ are$____

a. x, x^{-1}
b. x^L, x^{-L}
c. x^{2i}, x^{-2i}
d. x^2, x^{-2}

v) The singular point of the equation $(1 - x^2)y'' - 2xy' + 2y = 0$ are

a. 2, -2
b. 0,0
c. 1,-1
d. 3,-3

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