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SUBJECT CODE NO: - 2061
FACULTY SCIENCE AND TECHNOLOGY
B.Sc. T.Y. (Sem VI)
Examination March/April-2022 (To Be Held In June/July-2022)
Mathematics MAT-601
Real Analysis-II

[Time: 1:53 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1 A) Attempt any one. 08

a) Let $\mathcal{F}$ be any nonempty family of open subsets of a metric space M. then prove that $\bigcup_{G\epsilon\mathcal{F}} G$ is also an open subset of M.

b) If $\mathcal{F}$ is a family of closed subsets of a metric space M, then prove that $\bigcap_{G\epsilon\mathcal{F}} F$ is also closed.

B) Attempt any one 07

c) Define $\mathrm{d}: \mathrm{R} \times \mathrm{R} \rightarrow [0, \infty)$ by
$\mathrm{d}(x, x) = 0 \ \ (x \,\epsilon\, R)$
$d(x, y) = 1 \quad (x, y \,\epsilon R\,; \ \ x \neq y)$
then show that d is a metric for R.

d) Let f be a function from R^2 onto R^2 defined by $f(< x, y >) = x \ \ ; (< x, y \,\epsilon\, R^2)$ then show that f is continuous on R^2.

Q.2 A) Attempt any one. 08

a) Let $< M, P >$ be a metric pace and let A be proper subset of M. then prove that the subset G_A of A is open subset of $< A, P >$ if and only if there exists an open subset G_M. Of $< M, P >$ such that at $G_A = A \cap G_M$.

b) Let f be a bounded functions on [a, b]. then that every upper sum for f is greater than or equal to every lower sum for f.

B) Attempt any one. 07

c) If A is a connected subset of the metric space M, then prove that $\overline{A}$ is connected.

d) Find the Fourier series for the function $f(x) = x^2 \ \ in\, [-\pi, \pi]$.

Q.3 A) Attempt any one 05

a) If $< M, P >$ is a complete metric space and A is a closed subset of M, then prove that $< A, P >$ is also complete.

b) If each of the subsets E_1, E_2 .... Of R^1 is of measure zero, then prove that $\bigcup_{n=1}^{\infty} E_n$ is also of measure zero.

B) Attempt any one 05

c) Let $f(x) = x\ (0 \leq x \leq 1)$. Let σ be the subdivistion $\{0, \frac{1}{3}, \frac{2}{3}, 1\}$ of [0, 1]. Compute $U[f; 6]$.

d) Prove that every finite subset of any metric space is compact.

Q.4 Choose the correct alternative. 10

I) The discrete metric space $< R, d >$ is denoted by________.

a) R_d b) R^d c) R^1 d) R_0

II) If M is the closed interval [0, 1] with the absolute value metric, then $B\left[\frac{1}{4}; \frac{1}{2}\right]$ is the interval _________.

a) $\left[\frac{1}{4}, \frac{1}{2}\right]$ b) $\left[\frac{-1}{4}; \frac{1}{4}\right]$

c) $\left[\frac{-1}{2}; \frac{1}{2}\right]$ d) $\left[0, \frac{3}{4}\right]$

III) The metric space $< M, P >$ is said to be compact if $< M, P >$ is both_________.

a) Bounded and totally bounded
b) Complete and totally bounded
c) Connected and totally bounded
d) None of these

IV) The set of rational numbers is __________

a) Of measure zero b) Not of measure zero
c) Uncountable d) None of these

V) For all $n = 0, 1, 2 \ldots\ldots\ldots\ldots\ldots$.

$\int_{-\pi}^{\pi} \sin^2 nx\, dx = $ __________

a) 0 b) π c) $\frac{\pi}{2}$ d) π^2