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SUBJECT CODE NO:- 2062
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-VI)
Examination March/April-2022 (To Be Held In June/July-2022)
Mathematics MAT - 602
Abstract Algebra - II

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[Time: 1:53 Hours]   **[Max.Marks:50]**

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Please check whether you have got the right question paper.

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N.B   1) All questions are compulsory.
                2) Figures to the right indicate full marks.

Q.1   A) Attempt any one   08
        a) If V is the internal direct sum of $U_1, U_2, \text{------} U_n$, then prove that V is isomorphic to
            the external direct sum of $U_1, U_2, \text{------} U_n$.
        b) If $v_1, v_2, \text{-------}, v_n$ is a basic of a vector space V over F and if $w_1, w_2, \text{------} w_m$ in V
            are linearly independent over F, them prove that $m \leq n$

        B) Attempt any one: 07
        c) If F is a field of real numbers show that the set of real –valued continuous functions
            on the closed interval [0,1] forms a vector space over F.
        d) If F is the field of real numbers, show that the vectors (1,1,0) (3,1,3) and (5,3,3) in
            $F^{(3)}$ are linearly dependent over F,

Q.2   A) Attempt any one: 08
        a) Prove that if V is finite – dimensional vector space over F and $\vartheta \neq 0 \in V$, then there
            is an element $f \in \hat{V}$ such that $F(v) \neq 0$
        b) If V is the set of all continuous complex- valued functions on [0,1] with inner
            product defined by $(f(t), g(t)) = \int_0^1 f(t)\overline{g(t)} \, dt$, then prove that
            $$\left| \int_0^1 f(t)\overline{g(t)} \, dt \right|^2 \leq \int_0^1 |f(t)|^2 \, dt \int_0^1 |g(t)|^2 \, dt$$

        B) Attempt any one: 07
        c) If W is a subspace of a vector space V, then prove that A(W) is a subspace of $\hat{V}$.
        d) If $\{w_1, w_2, ---------, w_m\}$ is an orthonormal set in vector space V prove that
            $\sum_{i=1}^{m} |(w_i v)|^2 \leq \|V\|^2 \quad for \, any \, v \in V$.

Q.3   A) Attempt any one;   05
        a) If V is a vector space over a field F, then for $v \in V, \propto \in F$ prove that i) $\propto 0 = 0$,
            ii) $0v=0$, iii) $(-\propto)v = -(\propto v)$
        b) If a,b,c are real numbers such that a>0 and $a\lambda^2 + 2b\lambda + C \geq 0$ for all real number
            λ, then prove that $b^2 \leq ac$

        B) Attempt any one   05
        c) In an inner product space V over F, prove that
            $\|u + v\|^2 + \|u - v\|^2 = 2(\|u\|^2 + \|v\|^2)$

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d) If λ is a left – ideal of R and if M is an R-module, show that for $m \in M, \lambda m = \{xm + x \in \lambda\}$ is a submodule of M.

Q.4   Choose the correct alternative: 10
   i)   If V is a vector space and W is a subspace of V, then the vector space V/W is called- ------
         a) Null space   b) linear space   c) quotient space   d) none of these

   ii)   In an n-dimensional vector space, each set consisting of n+1 or more elements is -------
         a) Linearly independent   b) linearly dependent   c) basis   d) none of these

   iii) If W is a subspace of a finite dimensional vector space V, then dim W + dim A(W)=------
         a) $dim\hat{V}$ b) dim V   c) 0   d) none of these

   iv)   The norm of vector (1,-2,-3) is ---------
         a) 9   b) 4   c) $\sqrt{12}$ d) $\sqrt{14}$

   v)   In an inner product space V, if u is orthogonal to v then----------
         a) (u,v)=0   b) (u,v)=1   c) (u,v)=-1   d) none of these

2