Total No. of Printed Pages:02

SUBJECT CODE NO:- B-2021
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y. (Sem-I) Examination Oct/Nov 2019
Mathematics MAT - 101
Differential Calculus

[Time: 01:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
1) Attempt all questions.
2) Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) If $y = cosech^{-1}x$, then find $\frac{dy}{dx}$.

b) If u and v be two functions of x possessing derivatives of the n^{th} order, then prove that
$(uv)_n = u_n v + n_{C_1} u_{n-1} v_1 + n_{C_2} u_{n-2} v_2 + ---- + n_{C_r} u_{n-r} v_r + --- + n_{C_n} u\, v_n$

B) Attempt any one: 07

c) If $y = e^{ax} \cos^2 x \; sinx$, then find $\frac{d^n y}{dx^n}$.

d) If $y = x^2 \sin x$, prove that

$$\frac{d^n y}{dx^n} = (x^2 - n^2 + n) \sin\left(x + \frac{n\pi}{2}\right) - 2nx \cos\left(x + \frac{n\pi}{2}\right)$$

Q.2 A) Attempt any one: 08

a) If two functions f(x) and F(x) are derivable in a closed interval [a, b] and $F'(x) \neq 0$ for any value of x in [a,b] then prove that there exists at least one value 'c' of x belonging to the open interval (a, b) such that
$\frac{f(b)-f(a)}{F(b)-F(a)} = \frac{f'(c)}{F'(c)}$

b) If z = f(x,y) be a homogeneous function of x, y of degree n, then prove that
$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz, \forall\, x, y \;\epsilon$ the domain of the function.

B) Attempt any one: 07

c) Verify Rolle's theorem for the function $f(x) = (x-a)^m (x-b)^n$; m, n being positive integer , $x\, \epsilon [a, b]$

d) If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$; $x^2 + y^2 + z^2 \neq 0$,

Show that
$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

Q.3 A) Attempt any one:- 05

a) Prove that div $\vec{f}$ and curl $\vec{f}$ are point functions.

b) Prove that

$curl\left(\phi\overrightarrow{f}\right) = grad\ \phi x\ \overrightarrow{f} + \phi\ curl\ \overrightarrow{f}$

B) Attempt any one:- 05

c) Find grad ϕ if $\phi = 2x^2y^3 - 3y^2z^3$ at the point (1,-1,1)

d) If f is finitely derivable at c, then prove that f is also continuous at c.

Q.4 Choose the correct alternative: 10

i) For $x \in R$, cosh(-x) = ………

a) coshx b) –coshx c) sinhx d) –sinhx

ii) If y= sin(3x+5), then $y_3 = - - - -$

a) $3^2 \sin(3x + 5 + 3\frac{\pi}{2})$

b) $3^3 \sin(3x + 5 + 3\frac{\pi}{2})$

c) $3^3 \cos(3x + 5 + 3\frac{\pi}{2})$

d) None of these

iii) For $\forall\, x \in R,\ e^x = - - - - - -$

a) $1 + x + x^2 + - - - -$

b) $1 - x + x^2 - - - - -$

c) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + - - - -$

d) $1 + x + \frac{x^2}{2!} + - - - -$

iv) grad $(\vec{r}.\vec{a})$= ---------

a) 0 b) $\vec{a}$ c) $2\vec{a}$ d) $3\vec{a}$

v) If ϕ is constant then grad ϕ = --------

a) 0 b) 2 c) 1 d) -1