B-2029

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SUBJECT CODE NO:- B-2029
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y. (Sem-III) Examination Oct/Nov 2019
Mathematics MAT – 301
Number Theory

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.
i) Attempt all questions.
ii) Figures to the right indicate full marks.

Q.1 (A) Attempt any one 08

a) Prove that the linear congruence $ax \equiv b(mod\ n)$ has a solution if and only if d/b where $d = \gcd(a, n)$ If d/b, then show that it has d mutually incongruent solutions modulo n.
b) If p and q are distinct primes with $a^p \equiv a(mod\ q)$and $a^q \equiv a(mod\ p)$then prove that $a^{pq} \equiv a(mod\ pq)$.

(B) Attempt any one 07

c) Solve the set of simultaneous congruences
$$x \equiv 1(mod\ 3), x \equiv 2(mod\ 5), x \equiv 3(mod\ 7)$$

d) Find the remainder when 15! is divided by 17

Q.2 (A) Attempt any one 08

a) Show that τ and σ are both multiplicative functions.
b) If the integer n>1 has the prime factorization
$n = P_1^{K_1} P_2^{K2} \ldots\ldots\ldots P_r^{Kr}$ then prove
$$\emptyset(n) = n\left(1 - \frac{1}{P_1}\right)\left(1 - \frac{1}{P_2}\right)\ldots\left(1 - \frac{1}{P_r}\right)$$

(B) Attempt any one 07

c) Determine all solutions in positive integers for Diophantine equation $172x + 20y = 1000$.
d) Show that $18! \equiv -1(mod\ 437)$

Q.3 (A) Attempt any one 05

a) If $a = qb + r$ then prove that $\gcd(a, b) = \gcd(b, r)$
b) If P is prime and P/ab, then prove that P/a or P/b

(B) Attempt any one 05

c) By using Euler's theorem show that for any integer a,
$a^{37} \equiv a(mod\ 1729)$.

d) Calculate $\phi(5040)$

Q.4 Choose the correct alternatives 10

i) gcd (12378, 3054) is --------
a) 6 b) 4 c) 7 d) 8

ii) If for any two positive integers 3054 and 12378, gcd(3054, 12378)=6 then lcm (3054, 12378) is----------
(a) 630402 (b) 6400402
(c) 6300402 (d) 6500402

iii) If ca=cb(mod n), then $a \equiv b \left(\mathrm{mod}\ \frac{n}{d}\right)$ is true only if ---------
(a) d=gcd(a,n) (b) d = gcd(c,n)
(c) d=gcd(b,n) (d) d = gcd(a,b)

iv) Value of $\sigma(12)$ is ----------
a) 16 b) 27 c) 15 d) 28

v) Value of $\sum_{n=1}^{6} \tau(n)$ is ----------

a) 14 b) 12 c) 10 d) 8