B-2025

Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2025
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y. (Sem-V) Examination Oct/Nov 2019
Mathematics MAT - 501
Real Analysis – I

[Time: 1.30 Hours]   [Max.Marks:50]

Please check whether you have got the right question paper.
N.B
        1) All questions are compulsory
        2) Figures to the right indicate full marks

Q.1   a) Attempt any one
        i)   Prove that the set of all rational numbers is countable.   08
        ii)   If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L, then prove that   08
              $\{S_n\}_{n=1}^{\infty}$ cannot also converge to a limit distinct from L.

    b) Attempt any one :
        iii)   If B is an infinite subset of the countable set A, then prove that B is countable.   07
        iv)   Show that the sequence $\{\log(1/n)\}_{n=1}^{\infty}$ diverges to minus infinity   07

Q.2   a) Attempt any one
        i)   If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges, then prove that $\{S_n\}_{n=1}^{\infty}$ is a   08
              Cauchy sequence.
        ii)   Prove that any bounded sequence of real numbers has a convergent subsequence   08

    b) Attempt any one
        iii)   If $x = r\cos\theta, \; y = r\sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$ 07
        iv)   Prove that

$$\frac{\partial(y_1, y_2, -------, y_n)}{\partial(x_1, x_2, ------, x_n)} \cdot \frac{\partial(x_1, x_2, -----, x_n)}{\partial(y_1, y_2, -------, y_n)} = 1$$
    07

Q.3   a) Attempt any one
        i)   If $\sum_{n=1}^{\infty} a_n$ converges absolutely , then prove that $\sum_{n=1}^{\infty} a_n$ converges   05

        ii)   If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B, then prove that   05

            $\sum_{n=1}^{\infty}(a_n + b_n)$ converges to A+B

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b) Attempt any one
  iii) Find the values of x for which the series $\sum_{n=1}^{\infty} x^n/n$ converges absolutely   05
  iv) Prove that the series   05
      $\sum_{n=1}^{\infty}[1/n\,(n+1)]$ converges

Q.4 Choose the correct alternative and rewrite the sentence.   10
  i) If X_A is the characteristic function of set A then $X_{A \cup B} = - - - - - - - -$
     a) $X_A + X_B$
     b) $X_A + X_B + X_{A \cap B}$
     c) $X_A + X_B - X_{A \cap B}$
     d) $X_A - X_B + X_{A \cap B}$

  ii) The sequence $\{\sqrt{n}\}_{n=1}^{\infty}$ --------
     a) Diverges to minus infinity   b) converges to o
     c) converges to n   d) diverges to infinity

  iii) If $\{s_n\}_{n=1}^{\infty}$ where $S_n = (-1)^n$ then $\lim_{n \to \infty} \inf S_n = - - - - - -$
     a) 1   b) o   c) ∞ d) -1

  iv) If $\sum_{n=1}^{\infty} a_n$ is a series of real numbers and if $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty}|a_n|$
      diverges then $\sum_{n=1}^{\infty} a_n$ ----------
     a) Converges conditionally   b) Converges   c) oscillate   d) diverges to ∞

  v) If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty}(a_n + b_n) - - - -$
     a) Converges absolutely   b) converges conditionally   c) Oscillate   d) diverges

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