Total No. of Printed Pages:02

SUBJECT CODE NO:- B-2026
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y. (Sem-V) Examination Oct/Nov 2019
Mathematics MAT - 502
Abstract Algebra - I

[Time: 1:30 Hours] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) If H and K are subgroups of G, then prove that HK is a subgroup of group G if and only if HK=KH.

b) If $\emptyset$ is a homomorphism of G onto $\bar{G}$ with kernel K, then prove that K is normal subgroup of G.

B) Attempt any one: 07

c) If G is the group of all complex numbers a+ib, a, b are real, not both zero, under multiplication, and if $H = \{a + ib|a^2 + b^2 = 1\}$, then show that H is a subgroup of G.

d) Show that the intersection of two normal subgroups of G is also normal subgroup of G.

Q.2 A) Attempt any one: 08

a) Prove that the homomorphism $\emptyset$ of a ring R into a ring R' is an isomorphism if and only if $I(\emptyset) = (0)$, where $I(\emptyset)$ denotes the kernel of $\emptyset$

b) If $f(x), g(x)$ are two non-zero elements of the polynomial ring F[x], then prove that $\deg f(x) \cdot g(x) = degf(x) + degg(x)$

B) Attempt any one: 07

c) If R is a ring with unit element 1 and $\emptyset$ is homomorphism of R onto R', then prove that $\emptyset(1)$ is unit element of R'.

d) If R is the ring of all real valued continuous functions on interval [0,1] and if $M = \{f(x) \in R | f(\gamma) = 0 \; where \; 0 \le \gamma \le 1\}$, then prove that M is maximal ideal of R.

Q.3 A) Attempt any one:- 05

a) If G is a group then prove that the identity element of G is unique.

b) If p is prime number then prove that J_p, the ring of integers mod p is a field.

B) Attempt any one:- 05

c) If G is the group of integers under addition, H the subset consisting of all multiples of n, then show that H is subgroup of G.

d) If R and R' are any two arbitrary rings, where $R = R'$ and define $\emptyset: R \rightarrow R'$ by $\emptyset(a) = a$ for all $a \in R$ then show that $\emptyset$ is homomorphism. Also find the kernel of $\emptyset$.

Q.4 Choose the correct alternative and rewrite the sentence: 10

1) If every element of the group G is its own inverse then the group G is -----------
   a) Quotient group
   b) Normal subgroup
   c) Abelian group
   d) Non-abelian group

2) If $G = \{\pm 1, \pm i, \pm j, \pm k\}$ is a group of quaternions then $o(G) = -----$
   a) 0
   b) 2
   c) 4
   d) 8

3) If H is a subgroup of a group G, and if a, b ∈ G, then ----------
   a) $aH \neq bH \text{ and } aH \cap bH = \emptyset$
   b) $aH = bH \text{ or } aH \cap bH \neq \emptyset$
   c) $aH = bH \text{ or } aH \cap bH = \emptyset$
   d) $aH \neq bH \text{ and } aH \cap bH \neq \emptyset$

4) If $(R, +, \cdot)$ is a ring, then $(R, +)$ is ----------
   a) group
   b) Abelian group
   c) Commutator group
   d) finite group

5) Zero element of the quotient ring R/U is ----------
   a) R
   b) $R + U$
   c) $U + 1$
   d) U