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SUBJECT CODE NO:- B-2166
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem.-VI) Examination OCT/NOV 2019
Mathematics MAT - 602
Abstract Algebra – II

[Time: 1:30 Minutes] **[Max. Marks:50]**

Please check whether you have got the right question paper.

N.B
i. All questions are compulsory.
ii. Figures to the right indicate full marks.

Q.1 (A) Attempt any one: 08

(a) Prove that the kernel of a homomorphism T is a subspace of a vector space V, also that a homomorphism T is an isomorphism if and only if its kernel is (0).

(b) If $v_1, v_2, \dots, v_n$ are in a vector space V, then prove that either they are linearly independent or some v_k is linear combination of the preceding one $v_1, v_2, \dots, v_{k-1}$.

(B) Attempt any one: 07

(c) If V is finite-dimensional vector space and W is a subspace of V, then prove that there is a subspace W_1 of V such that $V = W \oplus W_1$.

(d) If U is a vector space and W a subspace of U, then prove that there is a homo morphism of U onto U/W.

Q.2 (A) Attempt any one: 08

(a) If $V = F^{(n)}$ $with$ $(u, v) = \alpha_1\bar{\beta}_1 + \alpha_2\bar{\beta}_2 + \cdots + \alpha_n\bar{\beta}_n$ where $u = (\alpha_1, \alpha_2, \dots . \alpha_n)$ and $v = (\beta_1, \beta_2, \dots ., \beta_n)$, then prove that

$$\left|\alpha_1\bar{\beta}_1 + \alpha_2\bar{\beta}_2 + \cdots + \alpha_n\bar{\beta}_n\right|^2$$
$$\le (|\alpha_1|^2 + |\alpha_2|^2 + \cdots + |\alpha_n|^2)(|\beta_1|^2 + |\beta_2|^2 + \cdots + |\beta_n|^2)$$

(b) If V is a finite-dimensional inner product space, then prove that V has an orthonormal set as basis.

(B) Attempt any one: 07

(c) In the vector space $F^{(n)}$ define for the vectors.

$$u = (\alpha_1, \alpha_2, \dots \ \alpha_n) \text{and } v = (\beta_1, \beta_2, \dots, \beta_n),$$
$$(u, v) = \alpha_1\bar{\beta}_1 + \alpha_2\bar{\beta}_2 + \cdots + \alpha_n\bar{\beta}_n,$$

then show that this defines as inner product on $F^{(n)}$.

(d) If F_2 is a family of polynomials of degree 2 at most. Define and inner product on $F_2 as$:

$$\big(p(x), q(x)\big) = \int_0^1 p(x)q(x)dx$$

If $\{1, x, x^2\}$ is a basis of the inner product space on F_2. Find an orthonormal basis from this basis.

Q.3 (A) Attempt any one 05

(a) If a, b, c are real numbers such tha $a > 0$ and $a\lambda^2 + 2b\lambda + c \geq 0$ for all real numbers $\lambda \geq 0,$ then prove that $b^2 \leq ac$.

(b) If $v_1, v_2, \ldots v_n \in V$ are linearly independent, then prove that every element in their span has a unique representation in the form $\lambda_1 v_1 + \lambda_2 v_2 + \cdots \ldots + \lambda_n v_n$ with the $\lambda_i \in F$.

(B) Attempt any one: 05

(c) If A and B are submodules of M, then prove that

$$A + B = \{a + b | a \in A, b \in B\}$$

is a submodule of M.

(d) If V is finite-dimensional vector space and T is a homomorphism of V into itself which is not onto, then prove that there is some $v \neq 0$ in V such that $T(v) = 0$.

Q.4 Choose the correct alternative and rewrite the sentence 10

1. A vector space with inner product is called…………
   (a) dual space
   (b) second dual space
   (c) inner product space
   (d) annihilator

2. If V is an inner product space over F, then for $v \in V, \alpha \in F$, we have $\|\alpha u\| =$………..
   (a) $\alpha^2\|u\|$
   (b) $\alpha\|u\|$
   (c) $\alpha\|\|u\|$
   (d) $|\alpha|\|u\|$

3. If W is a subspace of a vector space V over the field F, and if V/W is quotient space of W in V, then vector addition on V/W is defined as $(u + W) + (v + W) =$…………….. for all $u, v \in V$.
   (a) $(u + v) + W$
   (b) $(u - v) + W$
   (c) $u - v$
   (d) $u + v$

4. If V is a vector space over a field F, then the elements of F are called…….
   (a) Scalars
   (b) Vectors
   (c) linearly independent vectors
   (d) linearly dependent vectors

5. The norm of the vector (1, 0, -1) is ……………
   (a) 0
   (b) -1
   (c) 1
   (d) $\sqrt{2}$