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SUBJECT CODE NO:- B-2165
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem.-VI) Examination OCT/NOV 2019
Mathematics MAT-601
Real Analysis-II

[Time: 1:30 Minutes] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i. All questions are compulsory.
ii. Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) if (M, p) is a complete metric space for each $n \in I$, if F_n is a closed bounded subset of M such that $F_1 \supseteq F_2 \supseteq \cdots \supseteq F_n \supseteq F_{n+1} \supseteq \cdots$ and diam. $F_n \to 0$ as $n \to \infty$, then prove that $\bigcap_{n=1}^{\infty} F_n$ contains precisely one point.

b) If f is a continuous function from the compact metric space M_1 into a metric space M_2, then prove that the range $f(M_1)$ of f is also compact.

B) Attempt any one: 07

c) If l^{∞} denote the set of all bounded sequences of real numbers, if $x = \{x_n\}_{n=1}^{\infty}$ and $y = \{y_n\}_{n=1}^{\infty}$ are the points in l^{∞}, define $\rho(x, y) =_{1 \le n < \infty}^{\text{l.u.b}} |x_n - y_n|$, show that ρ is metric for l^{∞}.

d) If A and B are open subsets of R^1, then prove that $A \times B$ is a open subset of R^2.

Q.2 A) Attempt any one 08

a) If $f \in \Re[a, b]$ and $a < c < b$, then prove that $f \in \Re[a, c], f \in \Re[c, b]$ and also prove $\int_a^b f = \int_a^c f + \int_c^b f$

b) If the series $\frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ converges uniformly to $f(x)$ on $[-\pi, \pi]$, then prove that it is the Fourier series for $f(x)$ on $[-\pi, \pi]$.

B) Attempt any one: 07

c) Show that: $\lim_{n\to\infty} \frac{1}{n}\left[\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \cdots + \sin\frac{2\pi}{n}\right] = \frac{2}{\pi}$

d) Find the Fourier series expansion for $f(x) = |x|$ in $[-\pi, \pi]$.

Q.3 A) Attempt any one: 05

a) If A is subset of the metric space (m, ρ), and if (a, ρ) is compact, then prove that A is closed subset of (m, ρ).

b) If $f'(x) = 0$ for every x in the closed bounded interval [a,b], then prove that f is constant on [a,b].

B) Attempt any one: 05

c) If $f: R^2 \to R^2$ is defined by $f(x, y) = (y, x)$ for all $(x, y) \in R^2$, then show that f is continuous on R^2.

d) Prove that: $\frac{2\pi^2}{9} \le \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{2x}{\sin x} dx \le \frac{4\pi^2}{9}$.

Q.4 Choose the correct alternative and rewrite the sentence: 10

1) The discrete metric space (R, d) is denoted by ________.

a) R^d b) R_d c) R^1 d) l^∞

2) in a metric space M the subset E of M is closed if ________.

a) $E \subseteq \overline{E}$ b) $E = \phi$ c) $E = \overline{E}$ d) $E = R^1$

3) Union of infinite number of closed subsets of metric space_________.

a) Closed set b) Both open and closed set
c) Compact set d) need not be closed set

4) if f is continuous on [a,b], then threr exists $c \in (a, b)$such that $\int_a^b f(x)dx$ = __________.

a) $f(a)(b - c)$ b) $f(b)(c - a)$
c) $f(c)(b - a)$ d) $f(c)(b + a)$

5) For all n, $\int_{-\pi}^{\pi} \cos x dx$ =_________.

a) $-\pi$ b) π c) 0 d) 1