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SUBJECT CODE NO: - YY-2383
FACULTY OF SCIENCE AND TECHNOLOGY
B. Sc. (CBCGS) (Pattern 2022) F.Y SEM II
Examination April / May - 2024
Mathematics Paper -III Number Theory

[Time: 1:30 Hours]

[Max. Marks: 40]

Please check whether you have got the right question paper.

N. B

- 1) All the questions are compulsory.
- 2) Figures to right indicate full marks.



- Q1 A. Attempt any one: 05
- a. For integers a, b, c prove that
 - i. If a/b and b/c then a/c
 - ii. if a/b and a/c then $a/bx + cy$ for arbitrary integer's x and y .
 - b. For the fixed integer $h > 1$ and arbitrary integers a, b, c prove that
 - i. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then $a + c \equiv b + d \pmod{n}$
 - ii. If $a \equiv b \pmod{n}$ and $d/n, d > 0$ then $a \equiv b \pmod{d}$
- B. Attempt any one 05
- c. Find $\gcd(42823, 6409)$
 - d. Prove that any prime of the form $3k + 1$ is of the form $6k + 1$
- Q2 A. Attempt any one: 05
- a. If $(a, m) = (b, m) = 1$ then prove that $(ab, m) = 1$
 - b. State and prove that fundamental theorem of arithmetic.
- B. Attempt any one: 05
- c. Show that $61! + 1 \equiv 63! + 1 \equiv 0 \pmod{71}$
 - d. Solve that linear congruence $20x \equiv 4 \pmod{30}$
- Q3 A. (Attempt any one) 05
- a. If $F(n) = \sum_{d|n} f(d)$ for every positive integer n , then prove that $F(n) = \sum_{d|n} \mu(d)F(n/d)$
 - b. State and prove Chinese remainder theorem.
- B. Attempt any one. 05
- c. Find the smallest integer x for which $\phi(x) = 6$.
 - d. Solve the system of linear congruences $x \equiv 2 \pmod{3}, x \equiv 3 \pmod{5}, x \equiv 5 \pmod{2}$

Q4 Choose the correct alternatives:

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- i. For any positive integers a and b , if $\gcd(a, b) = 6$, and $\text{lcm}(a, b) = 60$, then _____
a. $ab = 90$ b. $ab = 60$ c. $ab = 180$ d. $ab = 360$
- ii. The value of x satisfying $4x \equiv 5 \pmod{9}$ is _____
a. 5 b. 6 c. 8 d. 9
- iii. value of $\sigma(12) =$ _____
a. 26 b. 28 c. 48 d. 12
- iv. value of $\Upsilon(12) =$ _____
a. 28 b. 5 c. 6 d. 4
- v. Value of $\phi(343) =$ _____
a. 342 b. 341 c. 146 d. None of these

