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SUBJECT CODE NO: - YY-2384
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. (CBCGS)(Pattern 2022) F.Y SEM II
Examination April / May - 2024
Mathematics Paper -IV Integral Calculus

[Time: 1:30 Hours]

[Max. Marks:40]

Please check whether you have got the right question paper.

- N. B
- (i) All questions are compulsory
(ii) Figures to the right indicates full marks

Q1 A) Attempt any one **05**

a) Obtain a reduction formula for $\int \cos^n x \, dx$ where 'n' is any integer.

b) Evaluate $\int \frac{1}{\sqrt{4x^2+1}} \, dx$

B) Attempt any one **05**

c) Evaluate $\int \frac{(x^2+1)}{(x^2-1)} \, dx$.

d) If 'n' is an even integer then write a formula for $\int_0^{\pi/2} \sin^n x \, dx$ Hence evaluate $\int_0^{\pi/2} \sin^6 x \, dx$

Q2 A) Attempt any one **05**

a) Evaluate $\int_a^b e^x \, dx$ as the limit of a sum

b) Show that the intrinsic equation of the semi-cubical parabola $3ay^2 = 2x^3$ is

$$S = \frac{4}{9} a (\sec^3 \psi - 1)$$

B) Attempt any one **05**

c) Prove that the length of an arc of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is $8a$.

- d) Find the volume of the solid generated by revolving the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ about the } x\text{-axis}$$

Q3 A) Attempt any one

- a) Prove that field \vec{F} is conservative if there exist a single valued differentiable scalar function ϕ such that $\vec{F} = \nabla\phi$
- b) If \vec{F} is a continuously differentiable vector point function and S is the surface bounded by a closed curve C, then prove that

$$\int_C \vec{F} \cdot d\vec{r} = \int_S (\text{curl } \vec{F}) \cdot \vec{n} \, ds$$

Where the unit normal vector \vec{n} at any point of S is drawn in the direction in which a right handed screw would move when rotated in the sense of description of C.

B) Attempt any one

- c) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = x^2y^2\vec{i} + y\vec{j}$ and the curve C is $y^2 = 4x$ in xy -plane from (0,0) to (4,4).
- d) Apply Gauss' divergence theorem to evaluate

$$\iiint_S [(x^3 - yz)dydz - 2x^2ydzdx + 2dx dy]$$

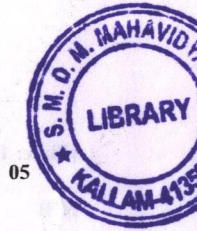
Over the surface of a cube bounded by the co-ordinate planes and planes

$$x = y = z = a.$$

Q4 Choose the correct alternative and rewrite the sentence

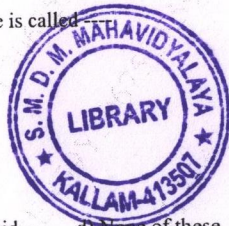
- (i) A curve $\vec{r} = \vec{f}(t)$, is called smooth if $\vec{f}(t)$, is -----

- a) Differentiable b) Continuously differentiable
c) Discontinuous d) None of these



(ii) The process of determining the length of the curve is called

- a) Quadrature b) Solid of revolution
c) Rectification d) None of these



(iii) $r = a(1 + \cos \theta)$ is an equation of -----

- a) Cycloid b) Cardioid c) Astroid d) None of these

(iv) The volume of the solid generated by the revolution about the x-axis of the area bounded by the curve $y = f(x)$, the ordinates at $x=a$, $x=b$ and the x-axis = _____

- a) $\Pi \int_a^b y^2 dx$ b) $\Pi \int_a^b x^2 dy$ c) $\int_a^b y^2 dx$ d) $\int_a^b x^2 dy$

(v) A vector point function is said to be irrotational in a region if its circulation along every closed curve in a region is -----

- a) 1 b) -1 c) 2 d) 0