Total No. of Printed Pages: 2

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SUBJECT CODE NO: - YY-2388
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. (CBCGS)(Pattern 2022) S.Y SEM III
Examination April / May - 2024
Mathematics-VI Laplace and Fourier Transforms

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[Time: 1:30 Hours]   [Max. Marks: 40]

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Please check whether you have got the right question paper.

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N. B   1) All questions are compulsory
                          2) Figures to the right indicate full marks

Q1   Attempt any five   10

   a) Prove that $\overline{n} = (n-1)!$ where n is positive integer

   b) Prove that $L\{\sin hat\} = \frac{a}{s^2 - a^2}$ where $S^2 > a^2$

   c) Find $L\{t \cos t\}$

   d) Define Laplace transform

   e) Find $L^{-1}\left\{\frac{1}{(s+2)^5}\right\}$

   f) Define Fourier cosine transform and Inverse Fourier cosine transform

   g) If F(s) is the complex Fourier transform of f(x) then show that
$$F\{e^{iax}f(x)\} = F(s+a)$$

Q2   A) Attempt any one   08
   a) If $L\{f(t)\} = F(s)$ then prove that

$$L\{f^{(n)}(t)\} = s^n L\{f(t)\} - s^{n-1}f(0) - s^{n-2}f'(0)$$
$$- \cdots - f^{(n-1)}(0);$$

$$\text{where} \quad f^{(n)}(t) = \frac{d^n}{dt^n}[f(t)]$$

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b) If $L^{-1}\{F(s)\} = f(t)$ then prove that $L^{-1}\{\int_s^\infty F(s)ds\} = \frac{f(t)}{t}$

07

B) Attempt any one

   c) Find $L\{t^3 e^{-2t}\}$

   d) Find $L^{-1}\left\{\frac{s-1}{s^2-6s+25}\right\}$

Q3 A) Attempt any one   **08**

   a) If F(s) is the complex Fourier transform of f(x) then show that
$$F\{f(x-a)\} = e^{isa} F(s)$$

   b) If $F(s)$ is the complex Fourier transform of f(x) then show that
$$F\{x^n f(x)\} = (-i)^n \frac{d^n}{ds^n}[F(s)]$$

B) Attempt any one   **07**

   c) Solve $\frac{dx}{dt} + y = 0$ and $\frac{dy}{dt} - x = 0$ under the conditions $x(0) = 1$ y(0)=0

   d) Find the Fourier sine and cosine transform of e^{-ax} where $a > 0$