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SUBJECT CODE NO: - Y-2028
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. (PATTERN-2013) (T.Y SEM V)
Examination April / May - 2024
Mathematics MAT - 501 Real Analysis - I

[Time:1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) Attempt all questions
2) All questions carry equal marks

Q.1 A. Attempt any one **08**

a) If A and B are equivalent and B and C are equivalent, then prove that A And C are Equivalent.

b) Show that a monotone sequence which is bounded is convergent.

B. Attempt any any one. **07**

c) Consider the function defined by $f(x) = \sin x. \quad (-\infty < x < \infty)$, then

i) What is the image of $\pi/4$ under f?

ii) What is the range of f?

iii) Find $\overline{f}^{1}(1)$,

iv) Find $f([0, \pi/6]), f([\pi/6, \pi/2]), f([0, \pi/2])$

d) Using definition of limit prove that $\lim_{n\to\infty} \frac{2n}{n+3} = 2$.

Q.2 A. Attempt any one. **08**

a) If $\{Sn\}_{n=1}^{\infty}$ is a sequence of real numbers which converges to L, then prove that $\{Sn^2\}_{n=1}^{\infty}$ converges to L^2.

b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

B. Attempt any one. **07**

c) For any a, b, ∈ R show that $||a|-|b|| \le |a-b|$. also if $\{Sn\}_{n=1}^{\infty}$ converges to L, then prove that $\{Sn\}_{n=1}^{\infty}$ converges to |L|

d) Prove that the series $2-2^{\frac{1}{2}}+2^{\frac{1}{3}}-2^{\frac{1}{4}}+$........diverges.

Q.3 Attempt any one. **5**

a) if $\{Sn\}_{n=1}^{\infty}$ and $\{tn\}_{n=1}^{\infty}$ are bounded sequences of real numbers, then Prove that,

$$\limsup_{n\to\infty}(s_n + t_n) \le \limsup_{n\to\infty} s_n + \limsup_{n\to\infty} t_n$$

b) If $y_1 = \gamma\cos\theta_1$, $y2 = \gamma \text{Sin}\theta_1 \text{Cos}\theta_2$, $y3 = \gamma \text{ Sin}\theta_1 \text{ Sin}\theta_2 \text{ Cos}\theta_3 \ldots y_{n-1}$ $=\gamma\ Sin\theta_1 \text{ Sin}\theta_2 \ldots . \text{Sin}\theta_{n-2} \text{Cos}\theta_{n-1}$ and $y_n = \gamma \text{ Sin}\theta_1 \text{ Sin}\theta_2 \ldots \text{Sin}\theta_{n-1}$ then Prove that.

$$\frac{\partial(Y_1\, y_2 \ldots y_n)}{\partial(\gamma, Q_1\, Q_2 \ldots Q_{n-1})} = \gamma^{n-1}\ Sin^{n-2}\theta_1, sin^{n-3}\theta_2 \ldots \text{Sin}\theta_{n-2}.$$

B) Attempt any one. **5**

c) If a Cauchy sequence has a sub- sequence converging to L then prove that Sequence itself converges to L.

d) Find the Jacobian $\frac{\partial(x,y,z)}{\partial(\gamma,\theta,\emptyset)}$ being given

$X = \gamma \text{ Cos}\theta\text{Cos}\emptyset$

$Y = \gamma \text{ Sin}\theta\sqrt{1-m^2 sin^2\emptyset}$

$Z = \gamma \text{ Sin}\emptyset\sqrt{1-n^2 sin^2\theta}$ where $m^2 + n^2 = 1$

Q.4 Choose the correct alternative: **10**

1) Which of the following set is not countable.
   a) Set of all integers.
   b) Set of Natural numbers.
   c) Set of all rational numbers.
   d) Sat of all irrational numbers.

2) Consider the statements
   i) Every subsequence of convergent sequence is convergent.
   ii) Every subsequence of a Cauchy sequence is Cauchy.
   a) Only (i) is true
   b) only (ii) is true
   c) Both (i) & (ii) are true.
   d) Both (i) & (ii) are false.

3) Let $\{s_n\}_{n-1}^{\infty}$ in a sequence of real number that is not bounded below then $\lim_{n\to\infty} inf\ s_n$=...
   a)-∞ b)∞ c)0 d)L>0

4) If x ≥1, then the series $\sum_{n=0}^{\infty} x^n$
   a) converges to zero
   b) converges to L≥0
   c) Diverges to infinity.
   d) Diverges to minus infinity.

5) If $u_1 u_2, u_2$, ...Un are functions of $x_1, x_2, x_3 \dots x_n$ and $x_1, x_2, x_3 \dots x_n$ are Functions of $y_1, y_2 \dots y_n$ then $\frac{\partial(u_1, u_2 \quad u_n)}{\partial(y_1, y_2, \quad y_n)}$ =.....
   a) $\frac{\partial(u_1,u_2,\dots\dots u_n)}{\partial(x_1 x_2,\dots\dots x_n)} \cdot \frac{\partial(x_1,x_2,\dots\dots x_n)}{\partial(u_1,u_2,\dots\dots u_n)}$
   b) $\frac{\partial(x_1,x_2,\dots\dots x_n)}{\partial(y_1,y_2,\dots\dots y_n)} \cdot \frac{\partial(y_1,y_2,\dots\dots y_n)}{\partial(u_1,u_2,\dots\dots u_n)}$
   c) $\frac{\partial(y_1,y_2,\dots\dots y_n)}{\partial(u_1,u_2,\dots\dots u_n)}$
   d) $\frac{\partial(u_1,u_2,\dots u_n)}{\partial(x_1,x_2,\dots\dots x_n)} \cdot \frac{\partial(x_1,x_2,\dots x_n)}{\partial(y_1,y_2,\dots\dots y_n)}$