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**SUBJECT CODE NO: - Y-2184**  
**FACULTY OF SCIENCE AND TECHNOLOGY**  
**B.Sc. T.Y. SEM VI (PATTERN-2013)**  
**Examination April / May - 2024**  
**Mathematics MAT-601 Real Analysis - II**

[Time: 1:30 Hours]

[Max. Marks: 50]

Please check whether you have got the right question paper.

- N. B
- i. All questions are compulsory.
  - ii. Figures to the right indicate full marks.

**Q1** (a) Attempt any one : **[08]**

- i. Show that every convergent sequence in a metric space is a Cauchy sequence.
- ii. Show that closure of a set is closed set.

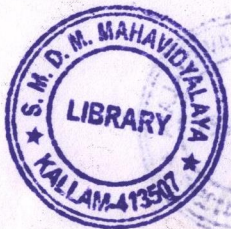
(b) Attempt any one : **[07]**

- i. If  $\tau : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow [0, \infty)$  is defined by  $\tau(P, Q) = \max(|x_1 - x_2|, |y_1 - y_2|)$ , where  $P = (x_1, y_1)$  and  $Q = (x_2, y_2)$ , then show that  $\tau$  is a metric on  $\mathbb{R}^2$
- ii. If  $f$  and  $g$  are continuous real-valued functions on the metric space  $M$ , and if  $A$  is the set of all  $x \in M$  such that  $f(x) < g(x)$ , then prove that  $A$  is open set.

**Q2** (a) Attempt any one: **[08]**

- i. Prove that the metric space  $(M, \rho)$  is compact if and only if every sequence of points of  $M$  has a subsequence converging to a point in  $M$ .
- ii. If  $f \in \mathcal{R}[a, b]$  and  $\lambda$  is any real number then  $\lambda f \in \mathcal{R}[a, b]$  and

$$\int_a^b \lambda f = \lambda \int_a^b f$$



(b) Attempt any one :

[07]

- i. Prove that the interval  $[0, 1]$  is not connected subset of  $\mathbb{R}_d$ .
- ii. If  $f(x) = x$ , ( $0 \leq x \leq 1$ ), and if  $\tau$  be the subdivision  $\left\{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right\}$  of  $[0, 1]$ . Compute  $U[f, \tau]$  and  $L[f, \tau]$ .

(a) Attempt any one:

[05]

- i. If  $f(x)$  is expanded in a series of cosines in the form

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx,$$

then prove that  $a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx$  and  $a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nxdx$

- ii. If  $f$  is continuous on the closed bounded interval  $[a, b]$  and if  $F(x) = \int_a^x f(t) dt$ , ( $a \leq x \leq b$ ), then prove that  $F'(x) = f(x)$  ( $a \leq x \leq b$ ).

(b) Attempt any one:

[05]

- i. Show that continuous function on compact metric space is uniformly continuous.
- ii. Find the Fourier series expansion for  $f(x) = |x|$  in  $[-\pi, \pi]$ .

4 Choose the correct alternative and rewrite the sentence

[10]

(a) In discrete metric space  $\mathbb{R}_d$ , the value of  $B\left[0; \frac{3}{2}\right] = \underline{\hspace{2cm}}$ .

- i.  $\{0\}$       ii.  $\left(-\frac{3}{2}, \frac{3}{2}\right)$       iii.  $\left[0, \frac{3}{2}\right]$       iv.  $\mathbb{R}_d$



(b) Consider the statements

( $\alpha$ )  $\mathbb{R}^1$  is complete metric space

( $\beta$ )  $\mathbb{R}^2$  is complete metric space, then \_\_\_\_\_

- i. only ( $\alpha$ ) is true
- ii. only ( $\beta$ ) is true
- iii. both ( $\alpha$ ) and ( $\beta$ ) are true
- iv. neither ( $\alpha$ ) nor ( $\beta$ ) is true

(c) If  $A$  is connected subset of the metric space  $M$ , and if  $A \subseteq B \subseteq \bar{A}$ , then \_\_\_\_\_

- i.  $\bar{B}$  is not connecte
- ii.  $\bar{B}$  is connected
- iii.  $\bar{B}$  is empty set
- iv.  $\bar{B}$  is disconnected



(d) If  $\tau^*$  is any refinement of  $\tau$  then \_\_\_\_\_

- i.  $L[f, \tau] \leq L[f, \tau^*]$
- ii.  $L[f, \tau] < L[f, \tau^*]$
- iii.  $L[f, \tau] \geq L[f, \tau^*]$
- iv.  $L[f, \tau] > L[f, \tau^*]$

(e) If we expand  $f(x)$  in the range  $(0, \pi)$  in a Fourier series of period  $2\pi$ , such expansion is called as \_\_\_\_\_

- i. full range Fourier series
- ii. double range Fourier series
- iii. triple range Fourier series
- iv. half range Fourier series