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SUBJECT CODE NO: - Y-2184
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y. SEM VI (PATTERN-2013)
Examination April / May - 2024
Mathematics MAT-601 Real Analysis - II

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
i. All questions are compulsory.
ii. Figures to the right indicate full marks.

Q1 (a) Attempt any one : **[08]**

i. Show that every convergent sequence in a metric space is a Cauchy sequence.

ii. Show that closure of a set is closed set.

(b) Attempt any one : **[07]**

i. If $\tau : \mathbb{R}^2 \times \mathbb{R}^2 \to [0, \infty)$ is defined by $\tau(P, Q) = \max(|x_1 - x_2|, |y_1 - y_2|)$, where $P = \langle x_1, y_1 \rangle$ and $Q = \langle x_2, y_2 \rangle$, then show that τ is a metric on $\mathbb{R}^2$

ii. If f and g are continuous real-valued functions on the metric space M, and if A is the set of all $x \in M$ such that $f(x) < g(x)$, then prove that A is open set.

Q2 (a) Attempt any one: **[08]**

i. Prove that the metric space $\langle M, \rho \rangle$ is compact if and only if every sequence of points of M has a subsequence converging to a point in M.

ii. If $f \in \mathcal{R}[a, b]$ and λ is any real number then $\lambda f \in \mathcal{R}[a, b]$ and

$$\int_a^b \lambda f = \lambda \int_a^b f$$

(b) Attempt any one : **[07]**

i. Prove that the interval $[0,1]$ is not connected subset of $\mathbb{R}_d$.

ii. If $f(x) = x$, $(0 \le x \le 1)$, and if τ be the subdivision $\left\{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right\}$ of $[0,1]$. Compute $U[f,\tau]$ and $L[f,\tau]$.

3 (a) Attempt any one: **[05]**

i. If $f(x)$ is expanded in a series of cosines in the form

$$f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos nx,$$

then prove that $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx$ and $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx$

ii. If f is continuous on the closed bounded interval $[a,b]$ and if $F(x) = \int_a^x f(t)dt$, $(a \le x \le b)$, then prove that $F'(x) = f(x)$ $(a \le x \le b)$.

(b) Attempt any one: **[05]**

i. Show that continuous function on compact metric space is uniformly continuous.

ii. Find the Fourier series expansion for $f(x) = |x|$ in $[-\pi, \pi]$.

4 Choose the correct alternative and rewrite the sentence **[10]**

(a) In descrete metric space $\mathbb{R}_d$, the value of $B\left[0; \frac{3}{2}\right] =$ _____.

i. $\{0\}$ ii. $\left(-\frac{3}{2}, \frac{3}{2}\right)$ iii. $[0, \frac{3}{2})$ iv. $\mathbb{R}_d$

(b) Consider the statements

(α) $\mathbb{R}^1$ is complete metric space

(β) $\mathbb{R}^2$ is complete metric space, then _____

i. only (α) is true

ii. only (β) is true

iii. both (α) and (β) are true

iv. neither (α) nor (β) is true

(c) If A is connected subset of the metric space M, and if $A \subseteq B \subseteq \bar{A}$, then _____

i. $\bar{B}$ is not connecte

ii. $\bar{B}$ is connected

iii. $\bar{B}$ is empty set

iv. $\bar{B}$ is disconnected

(d) If τ^* is any refinement of τ then _____

i. $L[f, \tau] \leq L[f, \tau^*]$

ii. $L[f, \tau] < L[f, \tau^*]$

iii. $L[f, \tau] \geq L[f, \tau^*]$

iv. $L[f, \tau] > L[f, \tau^*]$

(e) If we expand $f(x)$ in the range $(0, \pi)$ in a Fourier series of period 2π, such expansion is called as _____

i. full range Fourier series

ii. double range Fourier series

iii. triple range Fourier series

iv. half range Fourier series