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SUBJECT CODE NO: - Y-2185
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. (PATTERN-2013) (T.Y SEM VI)
Examination April / May - 2024
Mathematics MAT - 602 Abstract Algebra - II

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
i) All questions are compulsory.
ii) Figures to the right indicate full marks.

Q1 (a) Attempt any one: **08**

i) If S if non-empty subset of the vector space V, then prove that L(S), the linear span of S is subspace of V.

ii) Prove that the kernel of a homomorphism T is subspace of a vector space V and that T is an isomorphism if and only it its kernel is (0).

(b) Attempt any one : **07**

i) If U and W are two subspace of a vector space V, prove that
$$U+W=\{v\in V|v=u+w, u\in U, w\in W\}$$
is a subspace of V.

ii) Show that in $F^{(3)}$ the vectors (1, 1, 0), (3, 1, 3), (5, 3, 3) are linearly dependent.

Q2 (a) Attempt any one : **08**

i. If V is finite-dimensional vector space over F and $v(\neq 0)\in V$, then prove that there is an element $f\in\hat{V}$ such that $f(v)\neq 0$.

ii. If the system of homogeneous linear equations:
$$a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n=0,$$
$$a_{21}x_1+a_{22}x_2+\cdots+a_{2n}x_n=0,$$
$$a_{m1}x_1+a_{m2}x_2+\cdots+a_{mn}x_n=0$$
where $a_{ij}\in F$ is of rank r, then prove that there are n - r linearly independent solutions in $F^{(n)}$.

(b) Attempt any one : **07**

i. If F is the real field and V is $F^{(3)}$, inner product space, then show that Schwartz inequality implies that the cosine of an angle is of absolute value at most 1.

ii. If V is finite-dimensional vector space and T is a homomorphism of V into itself which is not onto, then prove that there is some $v\neq 0$ in V such that $T(v)=0$,

Q3 (a) Attempt any one: **05**

i. If W is a subspace of V over F, then prove that A(W) is subspace of $\hat{V}$.

ii. If W is subspace of inner product space V, then prove that $W^{\perp}$ is subspace of V.

(b) Attempt any one : **05**

i. If M is an R-module and if $m \in M$, and if

$$\lambda(m) = \{x \in R | xm = 0\}$$

Show that λ (m) is a left-ideal of R.

ii. If W_1 and W_2 are two subspaces of V, then prove that $W_1 \cap W_2$ is also subspace of V

Q4 Choose the correct alternative and rewrite the sentence **10**

(a) If S and T are subsets of a vector space V over F, then $L(S \cup T) =$

i. $L(S) - L(T)$ ii. $L(S) + L(T)$
iii. $L(S)L(T)$ iv. $L(S)/L(T)$

(b) If W is a subspace of a vector space V such that dim V = 5 and dim W = 2 then dim A(W) =

i. 5 ii. 2
iii. 3 iv. 7

(c) The additive identity 0 of a vector space V is called as _____

i. null vector or zero vector ii. perpendicular vector
iii. parallel vector iv. unit vector

(d) If T is a homomorphism of module M into a module N, then T is isomorphism if and only if ________

i. K(T) = T ii. K(T) = M
iii. K(T) = (0) iv. K(T) = N

(e) If V is a vector space over F, then the space Hom(V, F) is called

i. bidual space of F ii. a dual space of F
iii. bidual space of V iv. a dual space of V