Total No. of Printed Pages: 2

SUBJECT CODE NO: - Y-2039
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y (Sem-I)
Examination March / April - 2023
Mathematics MAT - 101 Differential Calculus

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B 1) Attempt all questions.
2) Figures to the right indicate full marks.

Q1 A) Attempt any one: 08

a) Show that f'(c), is the tangent of the angle which the tangent line to the curve y= f(x) at the Point P[c, f(c)] makes with x-axis.

b) If $y = e^{ax} \sin(bx + c)$, then show that $\frac{d^n y}{dx^n} = (a^2 + b^2)^{\frac{n}{2}} e^{ax} \sin(bx + c + n \tan^{-1}(\frac{b}{a}))$

B) Attempt any one: 07

c) If $y = \frac{x+1}{x^2-4}$; then find $\frac{d^n y}{dx^n}$

d) Find the value of the nth derivative of $y = e^m \sin^{-1} x$ for $x = 0$.

Q2 A) Attempt any one: 08

a) State and prove Cauchy's mean value theorem.

b) If z = f(x, y) be a homogeneous function of x, y of degree n then prove that
$x \frac{\partial Z}{\partial x} + y \frac{\partial Z}{\partial Y} = nz$,
$\forall$ x, y $\in$ the domain of the function.

B) Attempt any one: 07

c) Expand $2x^3 + 7x^2 + x - 6$ in Powers of $(x - 2)$.

d) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, x ≠ y, then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u \cos u$$

Q3 A) Attempt any one: 05

a) prove that:
curl $(\vec{f} \times \vec{g}) = \vec{f}\,\text{div}\,\vec{g} - \vec{g}\,\text{div}\,\vec{f} + (\vec{g}.\nabla)\vec{f} - (\vec{f}.\nabla)\vec{g}$.

b) Prove that:
grad $(\phi\psi) = \phi\,\text{grad}\,\psi + \psi\,\text{grad}\,\phi$.

B) Attempt any one: 05

c) Find grad $log|\vec{r}|$, where $\vec{r} = x\vec{\imath} + y\vec{j} + z\vec{k}$.

d) Find div $\vec{f}$ and curl $\vec{f}$, where $\vec{f}$ = grad $(axy^2 + byz + cz^2x^3)$

Q4 Choose the correct alternative. 10

i) If $y = x|x|$, then value of $\frac{dy}{dx}$ at the origin= _____.

a) 1 b) x c) 0 d) 2x

ii) If $y = \log(ax + b)$, then $\frac{d^n y}{dx^n} =$ _____

a) $\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$ b) $\frac{(-1)^n n!a^n}{(ax+b)^{n+1}}$ c) $\frac{(-1)^n(n-1)!a^{n-1}}{(ax+b)^n}$ d) $\frac{(-1)^{n-1}(n-1)!a^{n+1}}{(ax+b)^n}$

iii) $\log(1 + x) =$_____.

a) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} +$ ____ $+ (-1)^{\frac{n}{2}} \frac{x^n}{n!} +$ _____

b) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} +$ ____ $+ \frac{(-1)^{n-1}x^n}{n} +$ ____

c) $1 + \frac{x^3}{3!} + \frac{x^6}{6!} + \frac{x^9}{9!} +$ ____ $+ \frac{x^{3n}}{(3n)!} +$ _____

d) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} +$ ____ $+ \frac{x^n}{n} +$ ____

iv) If f(x) = |x|, x ϵ [~1,1]. then f(x)_____

a) Satisfy conditions of Lagrange's mean value theorem.

b) Does not satisfy conditions of Rolle's theorem.

c) Satisfy conditions of Rolle's theorem.

d) Satisfy conditions of Cauchy's mean value theorem.

v) If ψ is a constant, then grad $\psi =$ _____

a) 0 b) 1 c) -1 d) $-\psi$