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SUBJECT CODE NO: YY-2346
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y (Sem. I)
Examination May / June - 2023
Mathematics -II Deferential Calculus

[Time:1.30 Hours]

[Max. Marks:40]

Please check whether you have got the right question paper.

N. B.

- i) Attempt all questions.
 ii) Figures to the right indicate.

Q1 A) Attempt any one:

05

- a) Prove that the interior of a Set is an open set.
 b) Prove that every convergent Sequence is bounded.

B) Attempt any one:

05

- c) Show that $\lim_{n \rightarrow \infty} \frac{2n-3}{n+1} = 2$.
 d) Show that the function defined by $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ continuous at $x = 0$.

Q2 A) Attempt any one:

05

- a) If two functions f, g defined on $[a, b]$ are continuous on $[a, b]$, derivable on $]a, b[$ and $g'(x) \neq 0$, for any $x \in]a, b[$ then prove that there exists at least one real

number c between a and b such that $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$.

- b) If $y = \log(ax + b)$, then prove that $\frac{d^n y}{dx^n} = \frac{(-1)^{n-1}(n-1)!}{(ax+b)^n} a^n$.

B) Attempt any one:

05

- c) If $y = \sin(\sin x)$, prove that $\frac{d^2 y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$.
 d) Show that the function $f(x) = x^2$ is derivable on $[0,1]$.

Q3 A) Attempt any one:

05

- a) If \vec{u} and \vec{v} are two vector point functions, then prove that $\text{div}(\vec{u} + \vec{v}) = \text{div} \vec{u} + \text{div} \vec{v}$.
 b) If $\nabla(\phi\psi) = (\nabla\phi)\psi + \phi(\nabla\psi)$.

B) Attempt any one:

05

- c) If $\phi(x, y, z) = x^3 + y^3 + z^3 - 3xyz$ and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then show that $\vec{r} \cdot \nabla\phi = 3\phi$.
 d) Find $\frac{d^n y}{dx^n}$ if $y = \cos x \cdot x^3$.

Q4 Choose the correct alternative.

- i) The sequence $s_n = 1 + (-1)^n$; $n \in \mathbb{N}$.
- a) Converges to 0 b) Converges to 1
c) Oscillates finitely d) Oscillates infinitely
- ii) If $y = (2x + 5)^3$, then $y_3 = \dots\dots$
- a) $24(2x + 5)$ b) $6(2x + 5)^2$
c) $12(2x + 5)$ d) $36(2x + 5)$
- iii) A vector whose curl is zero is called _____
- a) Solenoidal vector b) Irrotational vector
c) Axial vector d) Polar vector
- iv) Which of the following is not true?
- a) The set \mathbb{R} of real numbers is an open set.
b) The set \mathbb{Q} of rational numbers is an open set.
c) A non-empty finite set is not open.
d) The set \mathbb{R} of real numbers is closed.
- v) $\lim_{n \rightarrow \infty} \frac{\sqrt{4+x}-2}{x} = \dots\dots\dots$
- a) 2 b) $\frac{2}{3}$ c) $\frac{1}{2}$ d) $\frac{1}{4}$