Total No. of Printed Pages: 2

SUBJECT CODE NO: - Y-2054
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y (Sem-II)
Examination March / April - 2023
Mathematics MAT - 201 (Integral Calculas)

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B 1) Attempt all questions.
2) Figures to the right indicate full marks.

Q1 A) Attempt any one. 08

a) Obtain a reduction formula for $\int x^n e^{-x} dx$ and hence show that the improper integral $\int_0^\infty x^n e^{-x} dx = n!$ Where n is any positive integer.

b) Evaluate the definate integral

$$\int_0^{\pi/2} Sin^n x \, dx$$

where n is a positive integer

B) Attempt any one. 07

c) Evaluate $\int \frac{x^5 dx}{x^3 - 2x^2 - 5x + 6}$

d) Evaluate $\int \frac{(x^3+2)}{(x-1)(x-2)^3} dx$

Q2 A) Attempt any one. 08

a) Evaluate $\int_a^b \cos h\, 2x \, dx$ as the limit of sum.

b) Find the area of the region lying x-axis and included between the circle $x^2 + y^2 - 2ax$ and the parabola $y^2 = ax$.

B) Attempt any one 07

c) Find the length of the arc of the curve $y = \log \tan h \left(\frac{x}{2}\right)$ from $x = 1\ to\ x = 2$

d) Find the volume of the solid obtained by revolving one arc of the cycloid $x = a(\theta + sin\theta), y = a(1 + cos\theta)$ about $x - axis$.

Q3 A) Attempt any one 05

a) If $\vec{F}$ is any continuously differentiable vector point function ans S is a surface bounded by curve C, then prove that
$\int_c \vec{F}.d\vec{r} = \int_s curl\ \vec{F}.\vec{n}\ ds$, where the unit normal vector $\vec{n}$ at any point of s is drawn in the sense in which a right handed screw would move when rotated in the sense of description of C.

b) Prove that $\int_s \vec{n} \times (\vec{a} \times \vec{r}) ds = 2aV$.

B) Attempt any one. 05

c) Show that $\frac{1}{3}\int_s \vec{r}.d\vec{a} = V$
where V is the volume enclosed by the surface S.

d) Evaluate $\int_c \vec{F}.d\vec{r}$ where
$\vec{F} = (2x^2 + y^2)\,\vec{i} + (3y - 4x)\,\vec{j}$
around the triangle ABC whose vertices are A(0,0) , B(2,0) , C(2,1).

Q4 Choose the correct alternatives. 10

1) $\int \frac{dx}{(2x-3)^3} =$
a) $\frac{-1}{4(2x-3)^2}$ b) $\frac{1}{4(2x-3)^2}$ c) $\frac{1}{(2x-3)^2}$ d) $\frac{-1}{(2x-3)^2}$

2) $\int_0^{\pi/2} \sin^9 x\, dx$
a) $\frac{315}{128}$ b) $\frac{-128}{315}$ c) $\frac{128}{315}$ d) 0

3) Perimeter of the cardioid $r = a(1 + \cos\theta)$is _____
a) 2a b) 4a c)6a d)8a

4) The volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the initial line is _____
a) $\frac{\pi ab^2}{3}$ b) $3\pi ab^2$ c) πab^2 d) $\frac{4}{3}\pi ab^2$

5) If C be a closed curve then $\oint \vec{r}.d\vec{r} =$ _____
a) r b) r^2 c) $\frac{1}{r}$ d) 0