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SUBJECT CODE NO: - YY-2349
FACULTY OF SCIENCE AND TECHNOLOGY
B. Sc. F.Y (Sem-II)
Examination March / April - 2023
Mathematics Paper -III
Number Theory

[Time: 1:30 Hours]

[Max. Marks: 40]

Please check whether you have got the right question paper.

N. B

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1 A. Attempt any one:

- a. If $m > 0$, then prove that $[ma, mb] = m[a, b]$ 05
- b. If p is a prime, then prove that p/a or p/b . 05

B. Attempt any one:

- c. Find the greatest common divisor of 7469 and 2464. 05
- d. Prove that $n^3 - n$ is divisible by 6. 05

Q2 A. Attempt any one:

- a. If $ax \equiv ay \pmod{m}$ and $(a, m) = 1$, then prove that $x \equiv y \pmod{m}$. 05
- b. If p is a prime, then prove that $(p-1)! \equiv -1 \pmod{p}$ 05

B. Attempt any one

- c. Prove that if p is a prime and $a^2 \equiv b^2 \pmod{p}$, then prove that $p \mid a+b$ or $p \mid a-b$. 05
- d. Find all integers that satisfy simultaneously : $x \equiv 2 \pmod{3}$,
 $x \equiv 3 \pmod{5}$, $x \equiv 5 \pmod{2}$ 05

Q3 A. Attempt any one

- a. If x is real number, then prove that $[x] \leq x < [x] + 1$, $x - 1 < [x] \leq x$, $0 \leq x - [x] < 1$. 05

- b. For every positive integer n , $\sum_{d|n} \Phi(d) = n$ 05

B. Attempt any one

- c. Prove that $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$, n is positive integer. 05

- d. Find all integers x and y such that $147x + 258y = 369$ 05

Q4 Choose the correct alternative and rewrite the sentences:

10

1. If a and b are integers with $a > 0$ then there exist unique integers q and r such that $b = qa + r$, where
a. $0 \leq r \leq a$ b. $0 \leq r < a$ c. $0 < r \leq a$ d. $0 < r < a$
2. The product of any three consecutive integers is divisible by ____
a. 4 b. 5 c. 6 d. 7
3. If m is positive integer then $a \equiv b \pmod{m}$ if and only if ____
a. $m/a + b$ b. $m/a - b$ c. m/ab d. $m/ma + b$
4. If $d(n)$ denotes the ____ positive divisors of n , then $d(12) =$ ____
a. 28 b. 24 c. 12 d. 6
5. If μ is a Mobious function then $\mu(8) =$ ____
a. -1 b. 1 c. 0 d. 8