Total No. of Printed Pages: 3

SUBJECT CODE NO: - Y-2051
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y (Sem-III)
Examination March / April - 2023
Mathematics MAT - 302 Integral Transforms

[Time:1.30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 a) Attempt any one of the following: 08

i. $If\ L^{-1}\{f(s)\} = F(t)\ and\ L^{-1}\{g(s)\} = G(t)$, then prove that

$$L^{-1}\{f(s)g(s)\} = \int_0^t F(u)G(t-u)du.$$

ii. $If\ L\{F(t)\} = f(s), then\ for\ n = 1,2,3\ldots..,prove\ that$

$$L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s).$$

b) Attempt any one of the following: 07

i. If $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(e^{-x}x^n)$, then prove that $L\{L_n(t)\} = \frac{(s-1)^n}{s^{n+1}}, s > 1$.

ii. Using Heavi-side's expansion formula, find $L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\}$.

Q2 a) Attempt any one of the following: 08

i. If $F(x)$ has the Fourier transform f (s) , the prove that $F(x)\ cos\ ax$ has the Fourier transform $\frac{1}{2}f(s-a) + \frac{1}{2}f(s+a)$.

ii. If $L\{F(t)\} = f(s)$, then prove that $\lim_{t\to\infty} F(t) = \lim_{s\to 0} s\, f(s)$.

b) Attempt any one of the following: 07

i. Find the Fourier transform of

$$f(x) = \begin{cases} x^2, & if\ |x| < a, \\ 0, & if\ |x| > a \end{cases}$$

ii. Evaluate the integral $\int_0^\infty e^{-ax}x^{m-1}cos\ bxdx$.

Q3 a) Attempt any one of the following: 05

i. If $L\{F(t)\} = f(s)$, then prove that $L\{e^{at}F(t)\} = f(s-a)$.

ii. If $f(s)$ is the Fourier transform of $F(x)$, then prove that $\frac{1}{s}f\left(\frac{s}{a}\right)$ is the Fourier transform of $F(ax)$.

b) Attempt any one of the following: 05

i. Find the value of $L^{-1}\left\{\frac{1}{s-2}+\frac{2}{s+5}+\frac{6}{s^4}\right\}$

ii. Prove that $L\{te^{at}\ sin\ at\} = \frac{2a(s-a)}{(s^2-2as+2a^2)^2}$.

Q4 Choose the correct alternative and rewrite the sentence: 10

a) If $0 < l < 1\ then\ \Gamma(l)\Gamma(1-l) =$ ……..

i. $\frac{sin\ l\pi}{\pi}$

ii. $\frac{cos\ l\pi}{\pi}$

iii. $\frac{\pi}{sin\ l\ \pi}$

iv. $\frac{\pi}{cos\ l\ \pi}$

b) If $L\{F(t)\} = f(s)$ then $L\left\{\frac{F(t)}{t}\right\}$ =……

i. $\int_0^\infty f(u)du$

ii. $\int_1^\infty f(u)du$

iii. $\int_s^\infty f(u)du$

iv. $\int_{-\infty}^\infty f(u)du$

c) $L^{-1}\left\{\frac{1}{s^2}\right\}$ = …….

i. t^2
ii. t
iii. t^3
iv. 1

d) The finite Fourier sine transform of $f(x) = 1 \; for \; 0 < x < \pi$ is …….

i. $\frac{\pi(-1)^{s+1}}{s}$

ii. $\frac{\pi(-1)^{s-1}}{s}$

iii. $\frac{\pi(-1)^{s+1}}{s^2}$

iv. $\frac{1-(-1)^s}{s}$

e) $L\{cosh\ at\} =$ …….

i. $\frac{a}{s^2-a^2}$

ii. $\frac{s}{s^2-a^2}$

iii. $\frac{a}{s^2+a^2}$

iv. $\frac{s}{s^2+a^2}$