Total No. of Printed Pages: 03

SUBJECT CODE NO: - Y-2047
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-V)
Examination March / April - 2023
Mathematics MAT - 502
Abstract Algebra - I

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 A) Attempt any one: 08

a) If H is a subgroup of a group G, then for a, $b \in G$ prove that the relation $a \equiv b$ mod H is an equivalence relation.

b) If ϕ is a homomorphism of a group G into group $\bar{G}$ with Kernel K, then prove that K is a normal subgroup of G.

B) Attempt any one: 07

c) If G is a group in which
$(a.b)^i = a^i.b^i$
for three consecutive integers i, for all a, $b \in G$, show that G is abelian.

d) Let G be a group and g is a fixed element in G. Define $\phi: G \to G$ by $\phi(x) = gx9^{-1}$. Prove that ϕ is an isomorphism of G onto G.

Q2 A) Attempt any one: 08

a) If ϕ is a homomorphism of a ring R into ring $R'$with Kernel $I(\phi)$, them prove that
   i. $I(\phi)$ is a subgroup of R under addition.
   ii. If $a \in I(\phi)$ and $r \in R$, then both ar and ra are in $I(\phi)$

b) Prove that if R is a commutative ring with unit element whose only ideals are (0) and R itself, then R is a field.

B) Attempt any one: 07

c) Prove that any field is an integral domain.

d) If U and V are ideal of R, and if
$U + V = \{u + v \,/\, u \in U \text{ and } v \in V\}$
Prove that U+V is also an ideal.

Q3 A) Attempt any one: 05

a) If H and K are subgroups of a group G and $O(H) > \sqrt{O(G)}, O(K) > \sqrt{O(G)}$, then prove that $H \cap K \neq (e)$.

b) If R is a commutative ring with unit element 1 and R/U is quotient ring then prove that
   i. R/U is commutative
   ii. R/U has a unit element 1+U

B) Attempt any one: 05

c) If N and M are normal subgroups of a group G, prove that NM is also a normal subgroup of G.

d) If R is ring with unit element 1 and ϕ is a homomorphism of R into R' prove that $\phi(1)$ is the unit element of R'

Q4 Choose the correct alternative: 10

i. If N is normal subgroup of a group G such that 0(G)=6 and 0(N)=3, then $0(G/N)$ = _______
   a) 3
   b) 2
   c) 9
   d) 18

ii. For any two elements a and b of a group G, if $(a.b)^2 = a^2.b^2$, then G is ______
   a) Abelian group
   b) Quaternion group
   c) Quotient group
   d) None of these

iii. If G is a group and for $x \in G, 0(x) = n$ and $x^m = e,$ then ______

a) m=0
b) m divides n
c) n divides m
d) none of these

iv. If an integral domain D is of finite characteristic, then its characteristic is _____

a) A composite number
b) A prime number
c) Any integer
d) None of these

v. If M is a maximal ideal of a commutative ring R with unit element, then ____

a) R/M is a field
b) R/M is not a field
c) R is a field
d) None of these