Total No. of Printed Pages: 2

SUBJECT CODE NO: - Y-2046
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-V)
Examination March / April - 2023
Mathematics MAT - 501
Real Analysis – I

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1. Attempt all questions.
2. All questions carry equal marks.

Q1 A) Attempt any one: 08

a) If $f: A \to B$ and $X \subset B, Y \subset B$, then prove that $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$

b) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of nonnegative numbers and if $\lim_{n\to\infty} S_n = L$, then prove that $L \geq 0$

B) Attempt any one: 07

c) Show that set of all integers is countable

d) Prove that the sequence $\left\{\frac{10^7}{n}\right\}_{n=1}^{\infty}$ has limit zero.

Q2 A) Attempt any one 08

a) Show that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent

b) If $\sum_{n=1}^{\infty} a_n$ Converges to A and$\sum_{n=1}^{\infty} b_n$ converges to B, then prove that $\sum_{n=1}^{\infty}(a_n + b_n)$ converges to A+B. also if $C \epsilon R$, then prove that $\sum_{n=1}^{\infty} Ca_n$ converges to CA.

B) Attempt any one 07

c) Prove that every subsequence of a Cauchy sequence is a Cauchy sequence.

d) Prove that the series $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + ---$ is divergent.

Q3 A) Attempt any one 05

a) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers and if $\lim sup_{n\to\infty}\ S_n = L = \lim\inf_{n\to\infty} S_n$ where $L\epsilon R$, then prove that $\lim_{n\to\infty} S_n = L$

b) Prove that $\frac{\partial(y_1,y_2,---;y_n)}{\partial(x_1,x_2,---;x_n)} \times \frac{\partial(x_1,x_2,---;x_n)}{\partial(y_1,y_2,---;y_n)} = 1$

B) Attempt any one. 05

c) Define i) Bounded sequence ii) Convergent sequence iii) Diverges to infinity sequence iv) Diverges to minus infinity sequence v) Monotone sequence

d) Show that the function

$u = x + y - z, v = x - y + z,$

$w = x^2 + y^2 + z^2 - 2yz$

Are not independent of one another.

Q4 Choose the correct alternative: 10

1) If $f: A \to B\ and\ X \subset B\ then\ f^{-1}(X) =$ -------
   a) $\{a \in A | f(a) \in X\}$
   b) $\{a \in B | f(a) \in B\}$
   c) $\{a \in X | f^{-1}(a) \in A\}$
   d) None of these

2) What is the value of $N \in I$ such that $\left|\frac{2n}{n+3} - 2\right| < \frac{1}{7}, n \geq N$
   a) 13 b) 23 c) 32 d) 40

3) Consider the statements
   i) Every bounded sequence is convergent
   ii) Every subsequence of a Cauchy sequence of real number is convergent
   a) Only (i) is true
   b) Only (ii) is true
   c) Both (i) & (ii) are true
   d) Both (i) & (ii) are false

4) If $\sum_{n=1}^{\infty} a_n$ is a convergent series then $\lim_{n\to\infty} a_n = L$ then L = ------
   a) $L \neq 0$ b) $L < 0$ c) L = 0 d) L > 0

5) If $u_1 = \frac{x_2x_3}{x_1}, u_2 = \frac{x_1x_3}{x_2}, u_3 = \frac{x_1x_2}{x_3}$ the $J(u_1,u_2,u_3) =$ ------
   a) 1 b) 2 c) 4 d) 16