Total No. of Printed Pages: 3

SUBJECT CODE NO: - Y-2115
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-V)
Examination March / April - 2023
Ordinary Differential Equation -I 504

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figure to the right indicate full marks.

Q1 A) Attempt any one:

a) Suppose a and b are continuous functions an interval I. Let A be a function such that A'= a. 8

The function Ψ given by

$\Psi(x) = e^{-A(x)} \int_{xo}^{x} e^{A(t)}\, b(t)dt.$

Where x_o is in I, is a solution of the equation $y' + a(x)y = b(x)$ on I.

The function ϕ, given by $\phi, (x) = e^{-A(x)}$ is a solution of the homogeneous equation $y' + a(x)y = 0$.

Prove that if c is any constant,

$\phi = \Psi + c\,\phi$, is a solution of $y' + a(x)y = b(x)$.

b) Consider the equation $y' + ay = 0$, where a is a complex constant if c is any complex number. Prove that the function ϕ define by $\phi(x) = c\, e^{-ax}$ is a solution of the equation $y' + ay = 0$. 8

B) Attempt any one:

c) Find all solution of the equation 7

$y' - 2y = x^2 + x.$

d) Consider the equation $Ly' + Ry = Ee^{iwx}$, where L,R,E,W are positive constant. Find the solution ϕ which satisfies $\phi(0) = 0$. 7

Q2 A) Attempt any one:

a) If ϕ_1, ϕ_2 are two solution of $y'' + a_1y' + a_2y = 0$ on an interval I containing a point x_o then prove th 8

$$w(\phi_1, \phi_2)(x) = e^{-a1(x-x_0)}.w(\phi_1, \phi_2)(x_0)$$

b) Let a_1, a_2 be constants and consider the equation $L(y) = y" + a_1 y' + a_2 y = 0$. If r_1, r_2 are distinct roots of the characteristics polynomial P, where $P(r) = r^2 + a_1 r + a_2$ then prove that the function ϕ_1, ϕ_2 define by $\phi_1(x) = e^{r_1 x}, \phi_2(x) = e^{r_2 x}$ are solution of $L(y) = 0$. 8

B) Attempt any one:

c) Find all solutions of the equation
$y" - 2y' - 3y = 0, \quad y(0) = 0, \quad y'(0) = 1.$ 7

d) Determine whether the functions $\phi_1(x) = cosx, \quad \phi_2(x) = 3(e^{ix} + e^{-ix})$ are linearly dependent or independent. 7

Q3 A) Attempt any one:

a) If z_1 *and* z_2 are two complex numbers then prove that
$||z_1| - |z_2|| \le |z_1 - z_2|$ 5

b) Prove that $e^{i\theta} = cos\theta + isin\theta$. 5

B) Attempt any one:

c) If r is such that $r^3 = 1$ and $r \neq 1$. Prove that $1 + r + r^2 = 0$. 5

d) If $z = x + iy$, where x, y are real, show that $|e^z| = e^x$. 5

Q4 Choose the correct alternative of the following. 10

1) $\frac{1+i}{1-1} =$ ___

a) -1 b) 0 c) i d) None of these

2) All solution of the equation $y" = x^2. on -\infty < x < \infty$

a) $\phi(x) = \frac{x^4}{12} + Gx^2 + C_2$.

b) $\phi(x) = \frac{x^4}{12} + Gx + C_2$.

c) $\phi(x) = \frac{x^3}{12} + Gx + C_2$.

d) None of these.

3) If $\phi_1(x) = cosx, \phi_2(x) = sinx$ then $w(\phi_1, \phi_2)(x) =$……..

a) 0 b) 1 c) cosx+sinx d) None of these

4) All solutions of the equation $y'' - 4y = 0$.
   a) $\phi(x) = Ge^{4ix} + c_2e^{-4ix}$.
   b) $\phi(x) = Ge^{4x} + c_2e^{-4x}$
   c) $\phi(x) = Ge^{-4ix} + c_2e^{-4x}$
   d) None of these

5) $\phi(x) = sin2x\ is\ a\ solution\ of\ the\ equation.$
   a) $y'' + 4y = 0$
   b) $y'' - 4y = 0$
   c) $y'' + 2y = 0$
   d) None of these