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SUBJECT CODE NO: - Y-2061
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-VI)
Examination March / April - 2023
Mathematics MAT-601 Real Analysis-II

[Time: 1: 30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 A) Prove any one: 08

a. Prove that every open subset G of R' can be written $G = \cup I_n$, where I_1, I_2 .... Are a finite number or a countable number of open intervals which are mutually disjoint..

b. Let $< M_1, P_1 > and < M_2, P_2 >$ be metric spaces, and let $f: M_1 \to M_2$ Then prove that f is continuous on M_1 if and only if $f^{-1}(F)$ is closed subset of M_1 whenever F is a closed subset of M_2

B) Attempt any one 07

c. For $P < x_1, y_1 >$ and $< x_2, y_2 >$, define $\sigma(P, Q) = |x_1 - x_2| + +y_1 - y_2|$, show that σ is a metric for the set of ordered pairs of real numbers.

d. Let f be the function from $R^2 onto\ R^1$ defined by
$f(< x, y >) = x \quad (< x, y > \in R^2)$
show that f is contnuous on R^2

Q2 A) Attempt any one 08

a. Let $< M_1, P_1 >$ be a compact metric space if f is a continuous function from M_1 into a metric space $< M_2, P_2 >$, then prove that f is uniformly continuous on M_1

b. If f is continuous on the closed bounded interval [a, b], and if

$$F(x) = \int_a^x f(t)dt \quad (a \le x \le b),$$

Then prove that $F'(x) = f(x) \quad (a \le x \le b)$

B) Attempt any one 07

c. Prove that every finite subset of any metric space is compact.

d. Find the Fourier series for the function $f(x) = e^x\ in\ -\pi < x < \pi$

Q3 A) Attempt any one 05

a. if A is a closed subset of the compact metric space < M, P>, then prove that the metric space <A, P> is also compact.

b. If $f \in R\,[a, b], \quad g \in R\,[a, b], \quad and\ if\ f(x) \le g(x)$ almost every where $(a \le x \le b)$ then prove that $\int_a^b f \le \int_a^b g$

B) Attempt any one 05

c. Let $f(x) = x\ (0 \le x \le 1), Let\ \sigma\ be$ the subdivision $\{0, \frac{1}{3}, \frac{2}{3}, 1\}$ of [0,1] compute $L\ [f; \sigma]$

d. If $0 \le x \le 1$ show that

$$\frac{x^2}{\sqrt{2}} \le \frac{x^2}{\sqrt{1+x}} \le x^2$$

Q4 Choose the correct alternative 10

1. The function P defined by $p(x, y) = |x - y|$ is a metric for the set R of real numbers, then the metric space $< R, P >$ is denoted by ____
   a. Rd   b. R^d c. R^1 d. R^∞

2. Every singleton set in a discrete metric space Rd is _______
   a. Open   b. closed   c. open and closed   d. none of these

3. The metric space R^1 is -------
   a. Not complete
   b. Totally bounded
   c. Complete but not totally bounded
   d. Complete and totally bounded

4. If f is Riemann integrable function on [a,b] and a<c<b, then _______
   a. $\int_a^b f > \int_a^c f + \int_c^b f$
   b. $\int_a^b f < \int_a^c f + \int_c^b f$
   c. $\int_a^b f = \int_a^c f - \int_c^b f$
   d. $\int_a^b f = \int_a^c f + \int_c^b f$

5. When m=n, for n=0,1, 2 ….

   $$\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = \cdots$$

   a. 0   b. 1   c. $-\pi$ d. π