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FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-VI)
Examination March / April - 2023
Mathematics MAT - 602 Abstract Algebra - II

[Time: 1 :30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figure to the right indicate full marks.

Q1 A. Attempt any one: 08

a. If T is homomorphism of a vector space U onto a vector space V with kernel W, then prove that V is isomorphic to U/W

b. Prove that if $v_1, v_2, \dots v_n$ are in vector space V. then either they are linearly independent or some V_k is a linear combination of the preceding ones, $v_1, v_2, \dots v_{k-1}$

B. Attempt any one 07

c. Let F be the field of all real numbers and let V be set of all sequences

$$\{(a_!, a_2, \dots a_n, \dots), |a_i^2 \varepsilon F\}$$

$If\ U = \{(a_!, a_2, \dots a_n, \dots), EV \,\big|\, \sum_{i=1}^{\infty} a_i^2\ \ is\ finite\}$ then prove that U is a subspace of V.

d. If T is an isomorphism of vector space V onto vector space W, then prove that T maps a basis of V onto a basis of W.

Q2 A. Attempt any one: 08

a. If W is subspace of finite-dimensional vector space V over F, then prove that $A\big(A(W)\big) = W$.

b. Prove that if V is finite-dimensional inner product space, then V has an orthonormal set as a basis.

B. Attempt any one: 07

c. Let V be the set of all continuous complex-valued function on the closed unit interval [0,1]. If $f(t), g(t) \in V$, such that

$$(f(t), g(t) = \int_0^1 f(t)$$

Prove that this define an inner product on V.

d. If A and B are submodules of on R Modules M, then prove that $A + B = \{a + b\ |a \in A, b\ \in A\}$ is a submodule of M.

Q3 A. Attempt any one: 05

a. If W is a subspace of an inner product space V, then prove that $W^{\perp}$ is a subspace of V.

b. If V is vector space over F and $v_1, v_2, \ldots, v_n \in V$ are linearly independent then prove that every element in their linear span has a unique representation in the from $\lambda_1 V_1 + \lambda_2 V_2 +$ ___$V_n V_n$ with the $\lambda_i EF$.

B. Attempt any one: 05

c. If V is finite-dimensional and $W_1$and $W_2$are subspaces of V, describe $A(W_1 \cap W_2)$ in terms of $A(W_1)$and $A(W_2)$

d. If F is the field of real numbers, prove that the vectors (1, 1, 0, 0), (0,1, -1, 0) and (0, 0,0, 3) in $F^{(4)}$ are linearly independent over F.

Q4 Choose correct alternatives: - 10

1. In an inner product space V, the inequality $|(u, v)| \leq ||u||$.‖v‖ is called ____
   a. Triangle inequality
   b. Bessel's inequality
   c. Schwarz inequality
   d. none of these

2. If V is a finite decisional vector space and $\hat{v}$ is its dual space then _____
   a. Dim $\hat{v} = sim\ V$ b. Dim $\hat{v} > sim\ V$ c. Dim $\hat{v} < sim\ V$ d. none of these

3. A subset S of a vector space V over F form basis if S is linearly independent and ___
   a. L(S)=S b. L(S)=V c. L(S)=F d) none of these

4. Every subspace of a vector space V other than (0) and V is called _______
   a. Improper subspace b. proper subspace c. dual space d. none of these

5. Vector space is defined over a _______
   a. Monoids b. group c. ring d. field