Total No. of Printed Pages: 03

SUBJECT CODE NO: - Y-2123
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-VI)
Examination March / April - 2023
Ordinary Differential Equation-II - MAT- 604

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 A) Attempt any one: 08

a) Let $\phi_1, \phi_2, \ldots\ldots, \phi_n$ be the n solutions of
$L(y) = y^n + a_1\, y^{(n-1)} + --- + a_n(x)y = 0$ on I satisfying
$\phi_i^{(i-1)}(x_0) = 1 \;, \phi_i^{(j-1)}(x_0) = 0 \;,\; j \neq i$
Prove that ϕ is any solution of L(y)=0 on I, three are n constant $C_1, C_2 --- C_n$
Such that $\phi = C_1\,\phi_1 + C_2\,\phi_2 + --+C_n\phi_n$

b) Let $\phi_1, \phi_2 \ldots \;, \phi_n$ be n solutions of
$L(y) = y^{(n)} + a_1(x)y^{(n-1)} + --+ a_n(x)y = 0$ on Interval I , and let x_0 be any point in I then Prove that $W(\phi_1, \phi_2, \ldots., \phi_n)(x) = \exp\left[-\int_{x_0}^{x} a_1(+)dt\right]$
$W(\phi_1, \phi_2, \ldots.\phi_n)(x_0)$

B) Attempt any one 07

c) Consider the equation
$$y'' + \frac{1}{x}\; y' + \frac{1}{x^2}\, y = 0 \; for\; x > 0$$

I. Show that there is a solution of the form x^r, where r is constant.

II. Find two linearly independent solutions for $x > 0$ and prove that they are linearly independent.

III. Find two solutions ϕ_1, ϕ_2 satisfying
$\phi_1(1) = 1 \quad , \quad \phi_2(1) = 0$
$\phi_1^1(1) = 0 \quad , \quad \phi_2^1(1) = 1$

d) Find all solutions of
$xy'' - (x+1)y' + y = 0$ given that one solutions is $\phi_1(x) = e^x (x > 0)$

Q2 A) Attempt any one 08

a) Let b be continuous on an interval I. Let $\phi_1, \phi_2, \dots, \phi_n$ be the basis for the solution of $L(y) = y^{(n)} + a_1(x)\, y^{(n-1)} + \dots + a_n(x) y = 0$

Prove that every solution ψ of $L(y) = b(x)$ can be written as:

$\psi = \psi_p + C_1\phi_1 + C_2\phi_2 + \cdots + C_n\phi_n$

Where ψ_p is particular solution of $L(y) = b(x)$ and $C_1, C_2 \dots C_n$ are constants.

Every such ψ is solution of $L(y) = b(x)$

A particular solution $\psi(P)$ is given by

$$\psi_p = \sum_{k=1}^{n} \phi_k(x) \int_{x_0}^{x} \frac{W_k(+) b(+)}{W(\phi_1, \phi_2 \dots \phi_n)t}\, dt$$

b) Consider the second order Euler equation $x^2y'' + axy' + by = 0$ (a ,b constant) and polynomial q is given by $q(r) = r(r-1) + ar + b$

Prove that basis for the solution of Euler equation on any interval not containing x = 0 is given by $\phi_1(x) = |x|^{r1}, \phi_2(x) = |x|^{r_2}$ in case r_1 & r_2 are distinct root of q.

B) Attempt any one: 07

c) Show that there is basis ϕ_1, ϕ_2 for the solution of

$xy'' + 4xy' + (2 + x^2)y = 0\ (x > 0)$

of the form $\phi_1(x) = \frac{\psi_1(x)}{x^2}, \phi_2(x) = \frac{\psi_2(x)}{x^2}$

d) Find the linearly independent power series solution of the equation $y'' - xy = 0$

Q3 A) Attempt any one : 05

a) Show that

$$\int_{-1}^{1} P_n(x)\, P_m(x) = 0 \quad (n \neq m)$$

b) Find all solutions of the equation $x^2y'' + 2xy^1 - 6y = 0 \quad (x > 0)$

B) Attempt any one 05

c) Find the singular point of the equation $x^2y'' + (x + x^2)\, y^1 - y = 0$ and determine those which are regular singular point.

d) Find all solutions ϕ of the form

$\phi(x) = |x|^r \sum_{k=0}^{\infty} C_k\, x^k\ (|x| > 0)$ for the equation

$x^2y'' + xy' + \left(x^2 - {}^1\!/_4\right) y = 0$

Q4 Choose the correct alternative 10

I. If $\phi_1, \phi_2 \dots \dots, \phi_n$ are n solutions of
$L(y) = y^{(n)} + a_1(x)\, y^{(n-1)} + \cdots a_n(x)y = 0$ an interval I, then they are linearly independent if and only if ……..

a) $W(\phi_1, \phi_2 \dots \dots \phi_n)(x) = 0 \ \forall x \in I$
b) $W(\phi_1, \phi_2 \dots \dots \phi_n)(x) \neq 0 \ \forall x \in I$
c) $W(\phi_1, \phi_2 \dots \dots \dots \phi_n)(x) = \exp\left[-\int_{x_0}^{x} a_1(+)dt\right]$
d) $W(\phi_1, \phi_2 \dots \dots \phi_n)(x) = \exp\left[\int_{x_0}^{x} a_1(+)dt\right]$

II. One solution of the equation $y'' - \frac{2}{x^2}\, y = 0\ (0 < x < \infty)$ is ………

a) $\phi(x) = x^2$
b) $\phi(x) = x$
c) $\phi(x) = e^x$
d) $\phi(x) = e^{-x}$

III. The singular point of the equation
$a_0(x)y^n + a_1(x)y^{(n-1)} + \cdots + a_n(x)y$ *is the point* x_0 *for which* ……

a) $a_0(x_0) \neq 0$
b) $a_1(x_0) = 0$
c) $a_0(x_0) = 0$
d) $a_1(x_0) \neq 0$

IV. $\int_{-1}^{1} P_n^2(x)dx =$ ………

a) $3/_{2n+1}$
b) $1/_{2n+1}$
c) $2/_{2n+1}$
d) $2/_{2n-1}$

V. The equation $x^2y'' + xy' + (x^2 - a^2)y = 0$ is

a) Euler equation
b) Legendre equation
c) Nonhomogeneous equation
d) Bessel equation